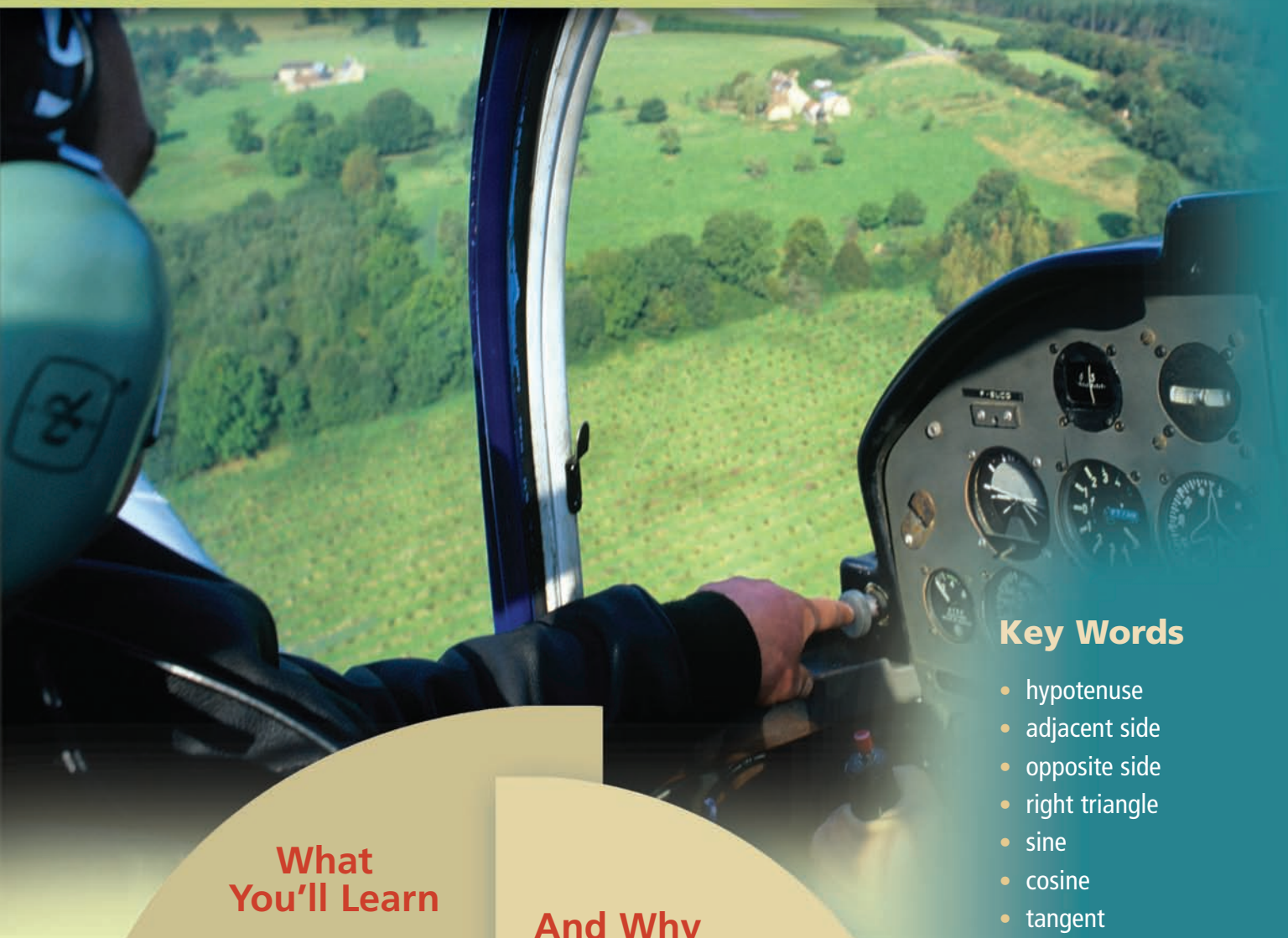
1 Trigonometry



What You'll Learn

To determine the measures of sides and angles in right triangles and acute triangles and to solve related problems

And Why

Real-world situations in surveying, navigation, construction, and design can involve determining the measures of sides and angles in triangles.

Key Words

- hypotenuse
- adjacent side
- opposite side
- right triangle
- sine
- cosine
- tangent
- primary trigonometric ratios
- acute angle
- angle of elevation
- angle of inclination
- angle of depression
- sine law
- cosine law

Activate Prior Knowledge

Metric Units of Length

Prior Knowledge for 1.1

The metre is a unit of length in the metric system.
The metric system of units is based on powers of 10.

1 cm = 10 mm 1 m = 100 cm 1 km = 1000 m
1 mm = 0.1 cm 1 cm = 0.01 m 1 m = 0.001 km

Example

The longest cartoon strip in *Guinness World Records 2006* was made by a group of 596 artists. The strip measures 238.4 m.

a) What is this length in centimetres?

b) What is this length in kilometres?

Solution

a) Since 1 m = 100 cm, multiply by 100 to convert to centimetres.
$238.4 \times 100 = 23\,840$
The cartoon strip is 23 840 cm long.

b) Since 1 km = 1000 m, divide by 1000 to convert to kilometres.
$238.4 \div 1000 = 0.2384$
The cartoon strip is 0.2384 km long.

Check

1. Convert each measure to the unit indicated.

a) 4560 m to kilometres
b) 156 cm to metres
c) 2.5 km to metres
d) 7.2 cm to millimetres
e) 34.5 m to kilometres
f) 9.81 mm to metres

2. Which metric unit would you use for each measure? Explain each choice.

a) Length of your pencil
b) Length of a bedroom
c) Thickness of a piece of wire
d) Distance from North Bay to Toronto

3. Joanna and Kush want to create a banner equal to the length of the gym.
They measure the length of the gym as 2600 cm.
The fabric is sold by the metre. What length of fabric do they need?

4. Katie uses a square sheet of paper with side length 7 cm to roll up 50 pennies. Each penny is 1.45 mm thick.
Can Katie roll up all the pennies? Justify your answer.

Imperial Units of Length

Prior Knowledge for 1.1

The imperial units of length are still in use in many industries.
Inch (in.), foot (ft.), yard (yd.), and mile (mi.) are units of length in the imperial system.

1 ft. = 12 in. 1 yd. = 3 ft.
1 mi. = 1760 yd. 1 mi. = 5280 ft.
A symbol for foot is ′ and for inch is ″.
A height of 6 ft. 6 in. is written 6′ 6″.

Example

From a blueprint, a carpenter calculated the length of baseboard needed for a room.
The perimeter of the room, excluding the doorway, is 532 in.
What length of baseboard, sold by the foot, should the carpenter purchase?

Solution

Since 1 ft. = 12 in., divide 534 by 12 to convert to feet.
532 ÷ 12 = 44 R4
So, 532 in. = 44 ft. 4 in.
The carpenter should purchase 45 ft. of baseboard.

The length is rounded up to 45 ft. so that the baseboard is not shorter than 44 ft. 4 in.

Check

1. Convert each measure to the unit indicated.

a) 6′ to inches
b) 96 yd. to feet
c) 7 mi. to feet
d) 36′ to yards
e) 8 ft. 4 in. to inches
f) 3 yd. 1 ft. to feet
g) 7040 yd. to miles
h) 192 in. to feet
i) 5 mi. to yards

2. Which imperial unit would you use for each measure? Explain your choice.

a) Length of your classroom
b) Length of a sub sandwich
c) Penalty spot distance in soccer
d) Length of the St. Lawrence Seaway

3. Roger is replacing the liner of a chimney. The length of liner required is 33 ft.
Roger has 400 in. of stainless steel pipe for the liner.
Is this enough? Justify your answer.

4. Marlene uses a 2 ft. by 1 ft. rectangular tray to hold her potted plants.
The diameters of the top of the pots are 4″ and 6″.

a) How many 4″ pots can fit on this tray? How many 6″ pots fit?
b) Marlene puts five 4″ pots on one tray. How many 6″ pots fit in the remaining space? Show your work using a diagram.

Ratio

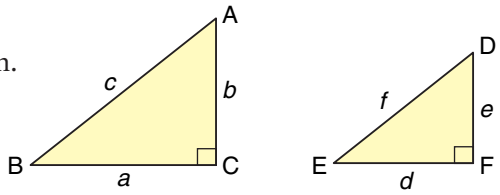
Prior Knowledge for 1.1

A **ratio** compares two quantities.

Example

ΔABC and ΔDEF are **similar triangles**.
In ΔABC, $a = 16$ cm, $b = 12$ cm, and $c = 20$ cm.
In ΔDEF, $d = 12$ cm, $e = 9$ cm, and $f = 15$ cm.
Determine the ratio for each pair of **corresponding sides**.
Are the ratios equivalent? Justify your answer.

a) $a:d$ **b)** $b:e$ **c)** $c:f$



Each vertex is labelled with a capital letter. Each side is labelled with the lowercase letter of the opposite vertex.

Solution

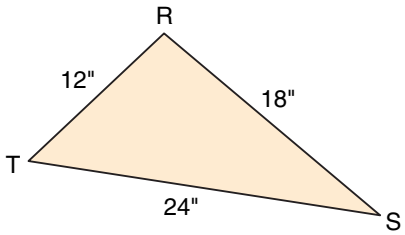
a) $a:d = 16:12 = 4:3$ **b)** $b:e = 12:9 = 4:3$ **c)** $c:f = 20:15 = 4:3$

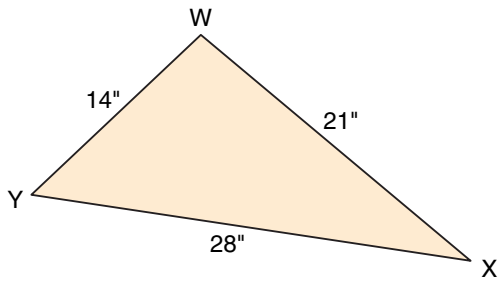
The ratios are equivalent because each ratio is equivalent to 4:3.

Check

1. ΔRST and ΔWXY are similar triangles.
Determine each ratio.

a) $r:w$ **b)** $s:x$ **c)** $t:y$





2. **a)** What do you notice about the ratios for the corresponding sides in question 1?

b) Make a prediction about the ratios for the corresponding sides of similar triangles. Justify your prediction.

3. For the Canadian flag, the ratio of width to length is 1:2.
What is the perimeter of a Canadian flag that is 12 ft. long?
Are all Canadian flags similar shapes? Explain using a diagram.

Solving Equations

Prior Knowledge for 1.1

To solve an equation for a variable, perform the same operation on each side of the equation to isolate the variable.

Example

Solve for x in each equation.

a) $6x = 12$ **b)** $\frac{x}{5} = 12$ **c)** $4.2 = 3.5x$

Solution

a) $6x = 12$ Divide each side by 6.

$\frac{6x}{6} = \frac{12}{6}$

$x = 2$

b) $\frac{x}{5} = 12$ Multiply each side by 5

$\frac{x}{5} \times 5 = 12 \times 5$

$x = 60$

c) $4.2 = 3.5x$ Divide each side by 3.5.

$\frac{4.2}{3.5} = \frac{4.2x}{3.5}$

$1.2 = x$

Check

1. Solve for x in each equation.

a) $20 = 8x$ **b)** $9 = \frac{x}{6}$ **c)** $39 = 0.75x$

2. George spent \$18.72 copying a story he wrote.
Photocopying costs \$0.08 per page. How many pages are in George's story?
Use an equation to solve this problem.

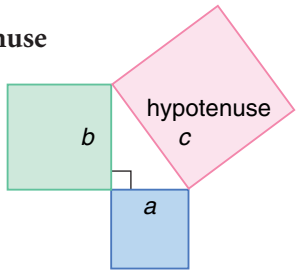
3. Raja has been offered two jobs.
Each of these jobs takes 24 weeks to complete.
One job pays \$3440 every 8 weeks. The other job pays \$2700 every 6 weeks.
Raja wants to accept the job that pays more per week.
Show how to use equations to help Raja make her choice.

4. Rashad pays \$6.30 tax for a pair of shoes that is priced at \$45.
His friend Zoe purchases another pair of shoes on sale for \$25 in the same store.
How much tax does Zoe have to pay?
Solve the problem in two ways. Show your work.
What assumptions did you make?

Pythagorean Theorem

Prior Knowledge for 1.1

The **Pythagorean theorem** relates the area of the square on the **hypotenuse** to the areas of the squares on the other two sides of a right triangle.



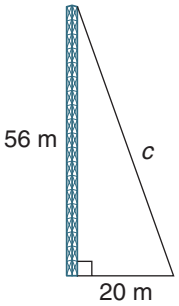
The Pythagorean theorem states that in a right triangle, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the lengths of the other two sides.

Example

A builder needs to purchase material for the guy wires of a 56-m tall tower. Each guy wire is anchored to the ground 20 m from the base of the tower. The material is sold by the metre. What is the length needed for each guy wire?

Solution

Sketch a diagram. Let the length of a guy wire be c.



Use $c^2 = a^2 + b^2$. Substitute: $a = 20$ and $b = 56$

$c^2 = 20^2 + 56^2 = 3536$

$c = \sqrt{3536}$

$c = 59.46$

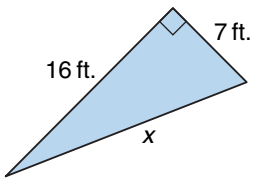
59.46 m is rounded to 60 m instead of 59 m to ensure there is enough material for each guy wire.

The length of material needed for each guy wire is 60 m.

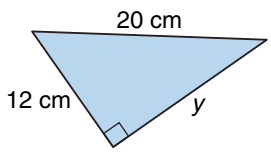
Check

1. Determine each unknown length.

a)

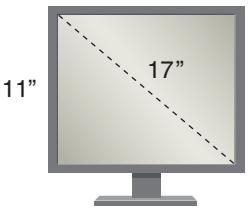


b)



2. Jadie uses an 18 ft. ladder to reach the window of the second floor of her house. She places the base of the ladder 4 ft. 6 in. away from the house to set the ladder at a safe angle. How far up the wall of the house is the second-floor window?

3. Corey purchased a 17″ flat screen for his computer. The height of the screen is 11″. Determine its width. Would the screen fit into a 14″ wide space? Explain.



4. A baseball diamond is a square with side length 90 ft. What is the distance between first base and third base? Describe the method you used to solve this problem.

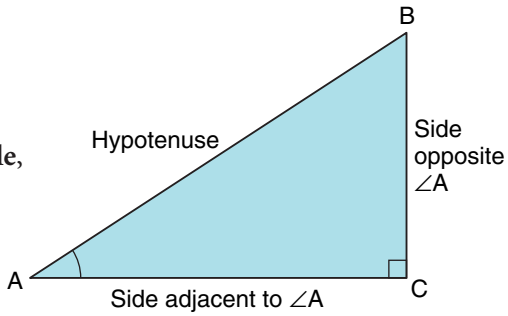
1.1 Sides of Right Triangles

Many fire departments have aerial ladder trucks. The ladder extends so that fire fighters can spray water on a fire or rescue people from upper floors.

Investigate Determining Sine, Cosine, and Tangent

Make sure your calculator is set to degrees for angle measures.

Work with a partner.
You will need a protractor and a scientific calculator.



- Each draw a large **right triangle**, and label it as shown.
- Measure the sides of your triangle to the nearest tenth of a centimetre.
  Measure the angles in your triangle to the nearest degree.
  Record the measurements on your triangle.

The sine of ∠A is written sin A.

- Copy the table. Calculate the ratios for the first 3 rows as decimals, to 2 decimal places.

If these keystrokes do not work for your calculator, check your User's Manual.

- To determine sin A, press SIN. Then enter the measure of ∠A and press) =. Record your answer in the table, to 2 decimal places. Continue for cos A and tan A.

Ratios	ΔABC
$\frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$	
$\frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$	
$\frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$	
sine of ∠A, or sin A	
cosine of ∠A, or cos A	
tangent of ∠A, or tan A	

Reflect

- How did you determine cos A and tan A?
- Compare your results with other pairs.
  What do you notice about the sine, cosine, and tangent of an angle in a triangle and the ratios of the triangle's side lengths?

Connect the Ideas

Primary trigonometric ratios

The **primary trigonometric ratios** are sine, cosine, and tangent. You can use these trigonometric ratios to calculate the length of a side of a right triangle if you know the measures of one **acute angle** and one side.

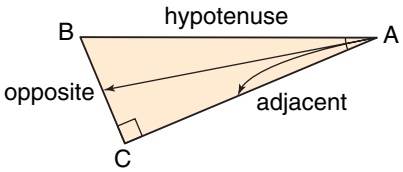
An acute angle is less than 90°.

When ∠A is an acute angle in a right triangle, then

$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

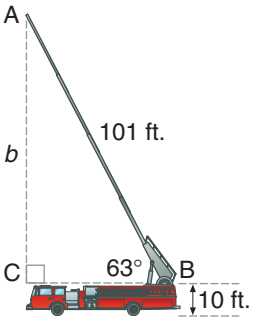
$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$$

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$



An **angle of elevation** is also called an **angle of inclination**.

A 10-ft. tall fire truck has an aerial ladder mounted on it. The ladder can extend to 101 ft. The **angle of elevation** of the ladder is 63°. How far up the building can the ladder reach, to the nearest foot?



From the diagram, the hypotenuse is 101 ft.
Determine the side length, b.
Since we know the length of the hypotenuse, c, use the sine ratio.

Use the sine ratio

Each vertex is labelled with a capital letter. Each side is labelled with the lowercase letter of the opposite vertex.

$$\sin B = \frac{\text{length of side opposite } \angle B}{\text{length of hypotenuse}}$$

Substitute: $\angle B = 63°$, $c = 101$

$$\sin 63° = \frac{b}{101}$$ Multiply each side by 101.

$$\sin 63° \times 101 = \frac{b}{101} \times 101$$

$$\sin 63° \times 101 = b$$ Press: SIN 63) × 101 =

$$b \doteq 89.99$$

Calculate the total distance

The base of the ladder is on top of the truck, which is 10 ft. above the ground. So, add 10 ft. to the height of the ladder.
$89.99 + 10 = 99.99$

The ladder can reach about 100 ft. up the building.

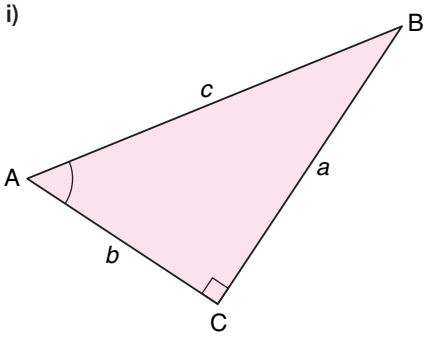
Practice

Give side lengths to the same number of decimal places as the given lengths.

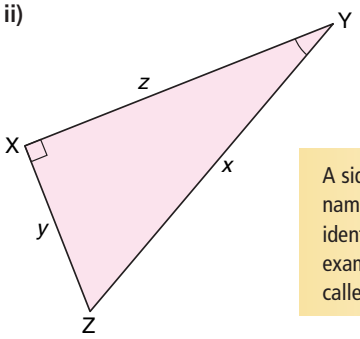
1. For each triangle, name these sides for the marked angle in one way with two capital letters, and in another way with one lower case letter.

a) hypotenuse **b)** opposite **c)** adjacent

i)



ii)

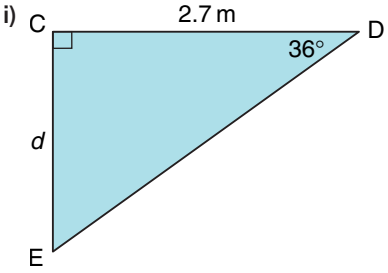


A side of a triangle can be named using the letters that identify its end points. For example, in ΔABC, side c can be called AB.

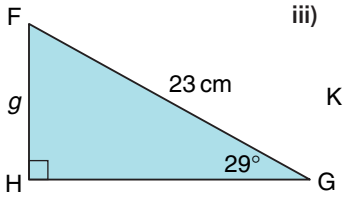
2. **a)** For each triangle, identify the hypotenuse, opposite side, and adjacent side for the angle whose measure is given.

b) For each triangle, which trigonometric ratio can you use to calculate the length of the indicated side? Explain your choice. Then, use the ratio to calculate the length.

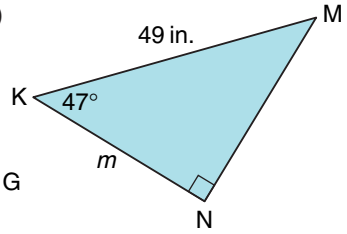
i)



ii)



iii)



3. Draw each triangle, then calculate.

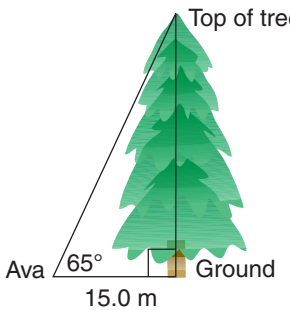
a) In ΔDEF, $\angle D = 57°$, $\angle F = 90°$, and $f = 150$ ft. Determine e.

b) In ΔXYZ, $\angle Y = 41°$, $\angle Z = 90°$, and $z = 4.5$ m. Determine y.

c) In ΔABC, $\angle B = 27°$, $\angle C = 90°$, and $a = 35$ cm. Determine b.

4. Ava's town is having a contest to find the tallest tree. To measure the height of a pine tree on her family's farm, Ava walks 15.0 m from the base of the tree. She measures the angle of elevation from the ground to the top of the tree as 65°. How tall is the tree to the nearest 10 cm?

1 m = 100 cm



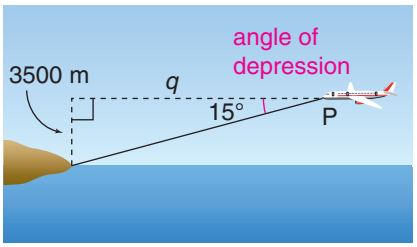
Sometimes, the unknown measure is the denominator of the ratio, not the numerator.

Example

A small plane is flying at an altitude of 3500 m over Lake Ontario, toward an island. The **angle of depression** to the island is 15°. How much farther, to the nearest kilometre, does the plane need to fly before it is above the island?

Solution

Let P represent the position of the plane. Let q represent the horizontal distance from the plane to the island in metres. Sketch a triangle. To determine the side length q, use the tangent ratio.



$$\tan \text{P} = \frac{\text{length of side opposite to } \angle\text{P}}{\text{length of side adjacent } \angle\text{P}}$$

$$\tan 15° = \frac{3500}{q}$$ Multiply each side by q.

$$\tan 15° \times q = \frac{3500}{q} \times q$$

$$\tan 15° \times q = 3500$$ To isolate q, divide each side by $\tan 15°$.

$$\frac{\tan 15° \times q}{\tan 15°} = \frac{3500}{\tan 15°}$$

$$q = \frac{3500}{\tan 15°}$$ Press: 3500 ÷ TAN 15) =

$$q \doteq 13\,062$$

1 km = 1000 m

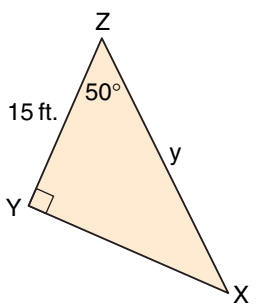
The distance the plane needs to fly before it is above the island is about 13 km.

5. In each diagram:

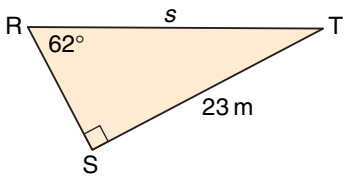
i) Which trigonometric ratio would you use to calculate the length of the labelled side? Explain your choice.

ii) Use the ratio to calculate the length to the nearest unit.

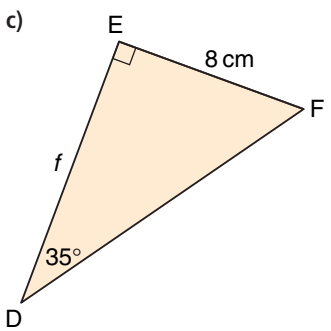
a)



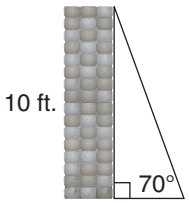
b)



c)



6. A carpenter is cutting a support brace to reach the top of a 10 ft.-high wall. She wants the brace to make an angle of 70° with the ground. How long should the brace be, to the nearest foot?

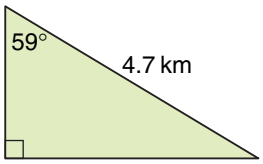


7. **Assessment Focus** A land surveyor draws a diagram of a plot of land that is in the shape of a right triangle. The longest side is 4.7 km long. The angle between the longest side and one other side is 59°.

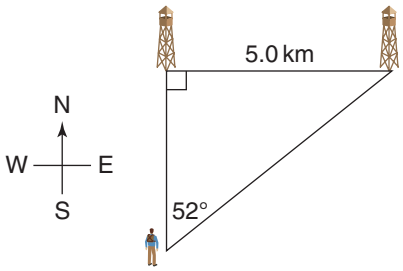
a) How long are the two shorter sides?

b) Describe another method to solve the problem.

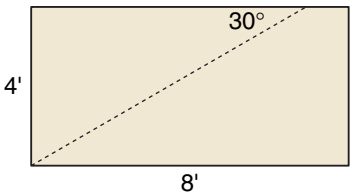
c) Which method do you prefer? Why?



8. A hiker can see a forest ranger tower due north of his position. He can see another tower that he knows is 5.0 km due east of the first tower. Using his compass, he faces the first tower, then turns 52° until he is facing the second tower. How far is he from each tower?



9. **Take It Further** Jovanna wants to cut a triangular piece off a 4′ by 8′ sheet of plywood. She wants the piece to have one side 4′ long, and a 30° angle in the corner opposite the 4′ edge.



a) How far along the 8′ side should she measure to mark the cutting line, to the nearest inch? Include a diagram.

b) Sketch another way Jovanna could cut the sheet.

In Your Own Words

The cosine of an acute angle in a right triangle has the same measure as the sine of the other acute angle in the triangle.
Explain why. Include a drawing with your explanation.

Literacy in Math

A Closer Look at Your Text

- Copy and complete the chart. Use checkmarks in the left columns to show whether you agree or disagree with each statement.

To start		Statement	Later	
Agree	Disagree		Agree	Disagree
		Math texts are mainly questions teachers assign for homework.		
		Math texts can be read just like any other book.		
		I use a math text only when instructed to do so by my teacher.		

- Compare your thoughts on these statements with a partner. Discuss reasons for any differences.

With your partner, complete this ***Text Quest***.

Text Quest

1. Look at the title page in a few chapters. List the features of the title pages.
2. This textbook has two indices. What are they? Why is each index useful?
3. Identify three types of technology addressed in the book.
4. Each chapter has a *Literacy in Math* feature. Identify three topics covered in these features that you think will be useful.
5. Look at *Connect the Ideas* in a few lessons. What is the purpose of the trapezoids?
6. In the *Mid-Chapter Review* and *What Should I Be Able to Do?*, there are red numbers in the margin. What do these numbers represent?
7. Choose something else in the text that you think can help you. Explain how it can help.

- Complete the right columns of your chart.
- Explain whether your thinking has changed. Why or why not?

1.2 Angles in Right Triangles

Mira is hiking due south of a park office. She knows there is a campsite due east of the park office.

To hike directly to the campsite, she needs to determine the direction to set her compass.
This requires determining an angle in a right triangle.

Investigate Determining Angles in Right Triangles

Work with a partner.
You will need a protractor and a scientific calculator.

The **legs** are the two shorter sides in a right triangle.

- Each draw a right triangle in which the ratio of one leg to the other leg is 3 to 5. For example, for one triangle, draw a right angle with one line segment 6 cm and the other, 10 cm. Join the endpoints to draw the hypotenuse. Label the dimensions. And for the other triangle, draw a right angle with one segment 9 cm and the other 15 cm.
- Measure each acute angle in your triangle. Compare measures of **corresponding angles** in the triangles. What do you notice?
- Press [2nd] [TAN]. Enter a tangent ratio for your triangles by pressing 3 [÷] 5. Press: [)] [=]. What do you notice?
- Press [2nd] [TAN]. Enter the other tangent ratio for your triangles by pressing 5 [÷] 3. Press: [)] [=]. What do you notice?
- Repeat this activity, using any ratio for the lengths of legs to draw another pair of triangles. Are the same relationships true?

If these keystrokes do not work for your calculator, check your User's Manual.

Reflect

- How do you know the ratio of one leg to the other in your first pair of triangles is 3 to 5?
- Explain how you determined the sides lengths for your second pair of triangles.
- Compare your results with other pairs.
  How does the tangent of an angle appear to be related to the lengths of sides in a right triangle?

Connect the Ideas

Determine an angle measure

When you press [2nd] [SIN], the calculator displays the measure of the angle whose sine is the number you enter.

For example, if you press: [2nd] [SIN] 0.75 [)] [=], the calculator displays 48.59. This shows that $\sin 48.59° \doteq 0.75$.

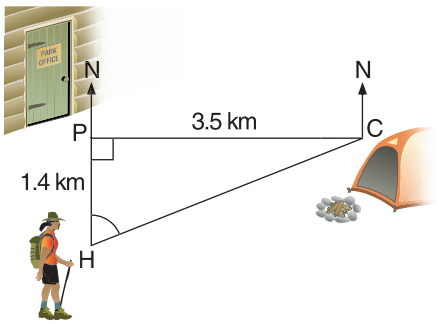
The [2nd] [COS] and [2nd] [TAN] keys work in the same way. These [2nd] function calculator keys represent **inverse operations**.

Mira is dropped off 1.4 km due south of the park office. The campsite is 3.5 km due east of the park office. To walk directly to the campsite, in what direction should Mira set her compass?

Mira's compass is marked at 2° intervals, so round the answer to the nearest 2°.
$\angle P$ is a right angle because the north and east directions intersect at 90°.

Draw a diagram

Determine the measure of $\angle H$.



The side opposite $\angle H$ is 3.5 km.
The side adjacent to $\angle H$ is 1.4 km.

Use the tangent ratio

Since you know the opposite and adjacent sides, use the tangent ratio.

$$\tan H = \frac{\text{length of side opposite } \angle H}{\text{length of side adjacent to } \angle H}$$

Substitute: length of side opposite $\angle H = 3.5$, length of side adjacent to $\angle H = 1.4$

$$\tan H = \frac{3.5}{1.4}$$

Press: [2nd] [TAN] 3.5 [÷] 1.4 [)] [=]

$$\angle H \doteq 68.2°$$

Clockwise is the direction that hands on a clock move.

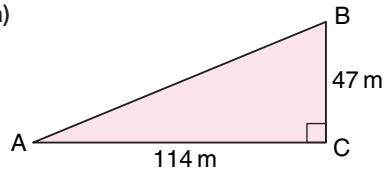
Mira should set her compass at 68°, or walk along a line that makes an angle of about 68° measured clockwise from north.

Practice

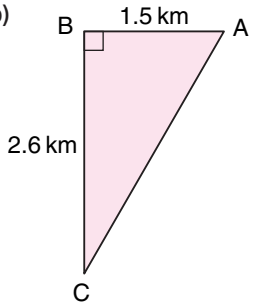
Give angle measures to the nearest degree.

1. For each triangle, determine tan A. Then, determine the measure of ∠A.

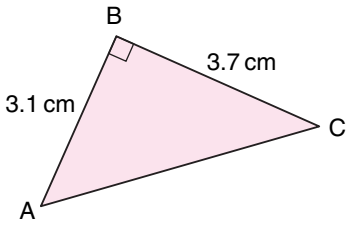
a)



b)

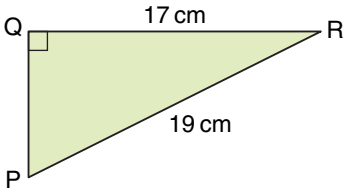


c)

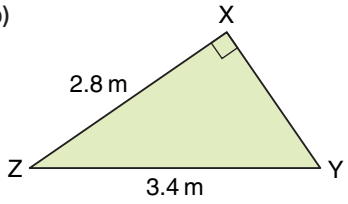


2. For the given information in each triangle, which angle can be calculated using the sine ratio? Determine its measure.

a)



b)



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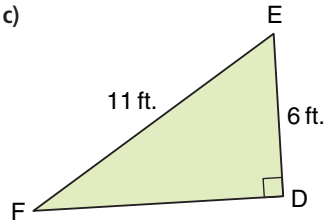
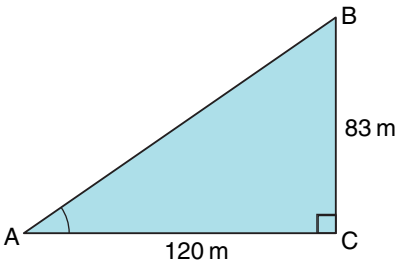
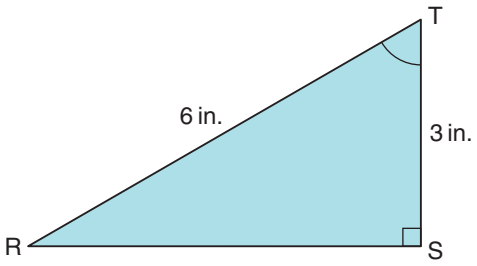


3. Decide which trigonometric ratio can be used to determine the measure of the marked angle in each diagram. Explain each choice. Then, use the ratio to determine the angle measure.

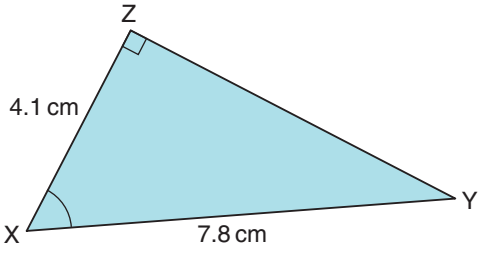
a)



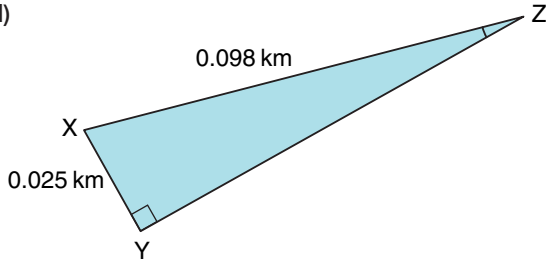
b)



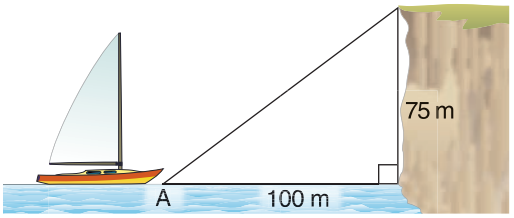
c)



d)

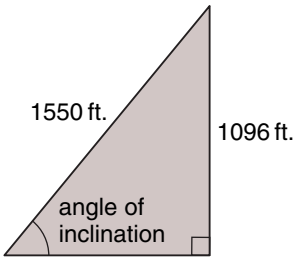


4. A boat is 100 m from the bottom of a 75-m high cliff.



Calculate the angle of elevation, ∠A, from the boat to the top of the cliff.

5. The Royal Gorge Bridge Incline Railway in Colorado is the steepest railway in the world. The elevation increases 1096 ft. over the 1550 ft. incline. Calculate the angle of inclination of the track.



6. Sketch each triangle, then calculate the angles.

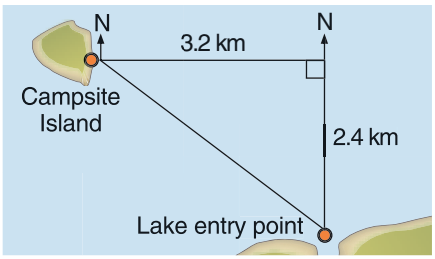
a) In ΔABC, $a = 1.2$ cm, $b = 4.5$ cm, and ∠C = 90°. Determine the measure of ∠B.

b) In ΔDEF, $d = 43$ yd., $f = 92$ yd., and ∠F = 90°. Determine the measure of ∠D.

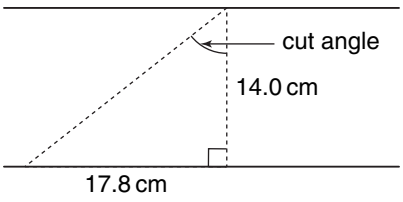
c) In ΔNPQ, $p = 49$ m, $q = 25$ m, and ∠P = 90°. Determine the measure of ∠N.

When you sketch, you do not need to draw to scale.

7. A canoeist uses a compass to navigate from a lake entry point to a campsite on a small island. The campsite is 2.4 km due north and then 3.2 km due west of the lake entry point. In what direction should the canoeist head to go directly to the campsite? Express your answer in degrees clockwise from north, rounded to the nearest 2°.



8. A carpenter is cutting a board that is 14.0 cm wide. She is using a radial saw that can be set to cut at any angle. To fit under a stairway, the board needs to be 17.8 cm shorter on one side than the other. At what angle should the carpenter make the cut, to the nearest degree?



Sometimes you need to determine the length of a side or convert a length to different units before you can use a trigonometric ratio to solve a problem.

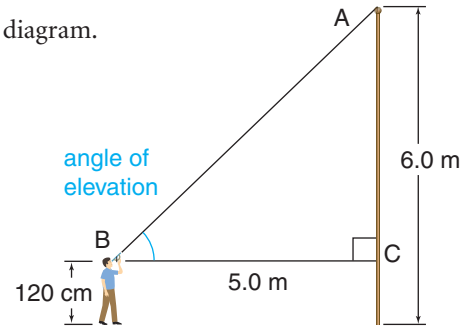
Example

A clinometer can be used to measure the angle of elevation or the angle of depression.

Enricho is checking the accuracy of his clinometer.
The school's flagpole is 6.0 m tall.
He walks 5.0 m from the base of the pole and holds the clinometer at eye level, 120 cm above the ground.
The clinometer shows the angle of elevation from Enricho's eye to the top of the flagpole as 45°. How accurate is the reading on the clinometer, to the nearest tenth of a degree?

Solution

Draw a diagram.



100 cm = 1 m, so, 120 cm = 1.2 m.

The flagpole is perpendicular to the ground, and the ground is parallel to the horizontal line of sight.
So, ΔABC is a right triangle with $\angle C = 90°$.
In ΔABC, the length of the side opposite $\angle B$ is: $6.0 \text{ m} - 1.2 \text{ m} = 4.8 \text{ m}$.
The side adjacent to $\angle B$ is 5.0 m.
Use the tangent ratio to calculate the angle of elevation:

$$\tan B = \frac{\text{length of side opposite } \angle B}{\text{length of side adjacent to } \angle B}$$

Substitute 4.8 and 5.0.

$$\tan B = \frac{4.8}{5.0}$$

Press: 2nd TAN 4.8 ÷ 5) =

$$\angle B \doteq 43.8°$$

The clinometer reading was 45°.
$45° - 43.8° = 1.2°$
The device is accurate to within about 1.2°.

9. A wheelchair ramp is 3.2 m long.
It rises from ground level to a porch that is 35 cm above the ground.
For safety reasons, there are maximum angles a wheelchair ramp can make with the ground.
Does this ramp meet either of the following rules?
Justify each answer. Include diagrams.

a) For power wheelchairs, the maximum angle is 7.1°.

b) For self-propelled wheelchairs, the maximum angle is 4.8°.

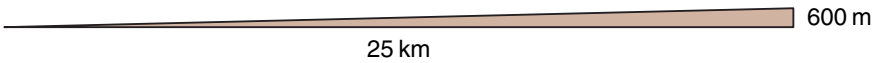
10. Assessment Focus A train track runs along a river in a valley.
The track rises a vertical height of 600 m over a horizontal distance of 25 km.

a) What is the angle of inclination of this section of the track, to the nearest tenth of a degree?

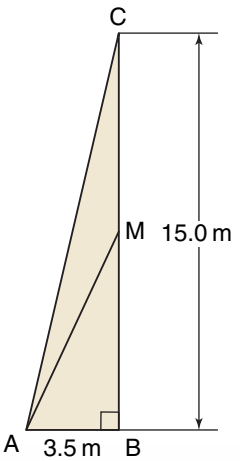
b) Create another problem for the diagram.
Solve your problem.

c) Explain your thinking as you created the problem.

d) Trade problems with a classmate to solve.
Compare solutions.



11. Take It Further There are two support wires on one side of a 15.0-m hydro pole.
Both wires are attached to the ground 3.5 m from the base of the pole.
One wire goes to the top of the pole.
The other wire goes to the middle of the pole.
What is the angle between the two wires where they meet at the ground?



In Your Own Words

Press: 2nd COS 12 ÷ 13) =.
What does the number in the display represent?
Illustrate your answer with a diagram.

1.3 The Sine Law

Many real-world situations involve triangles that do not contain right angles.

The primary trigonometric ratios only apply to right triangles.
However, there are other relationships between the sides and angles of triangles.

Inquire Relating Sides and Angles in Triangles

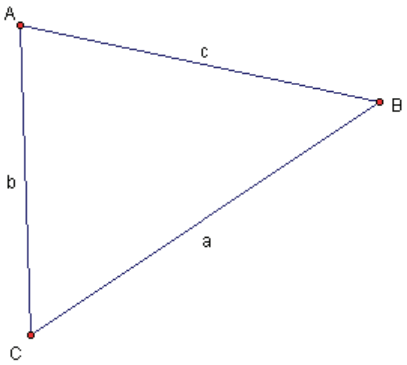
Choose Using *The Geometer's Sketchpad* or Making a Table.

Using The Geometer's Sketchpad

You will need *The Geometer's Sketchpad*.

- If you are using the file *Triangle.gsp*, open it and begin at question 9.
  The triangle on your screen should look like the one at the right.
- If you are not using the file *Triangle.gsp*, complete all the questions.

1. Start the program.
From the **Edit** menu, select **Preferences**.
Set **Angle** to be measured in degrees to the nearest unit.
Set **Distance** to be measured in centimetres to the nearest hundredth.



2. Use the **Point** tool.
Click on the screen in three places to construct three points.
Use the **Text** tool.
Click on each point to label the points A, B, and C.

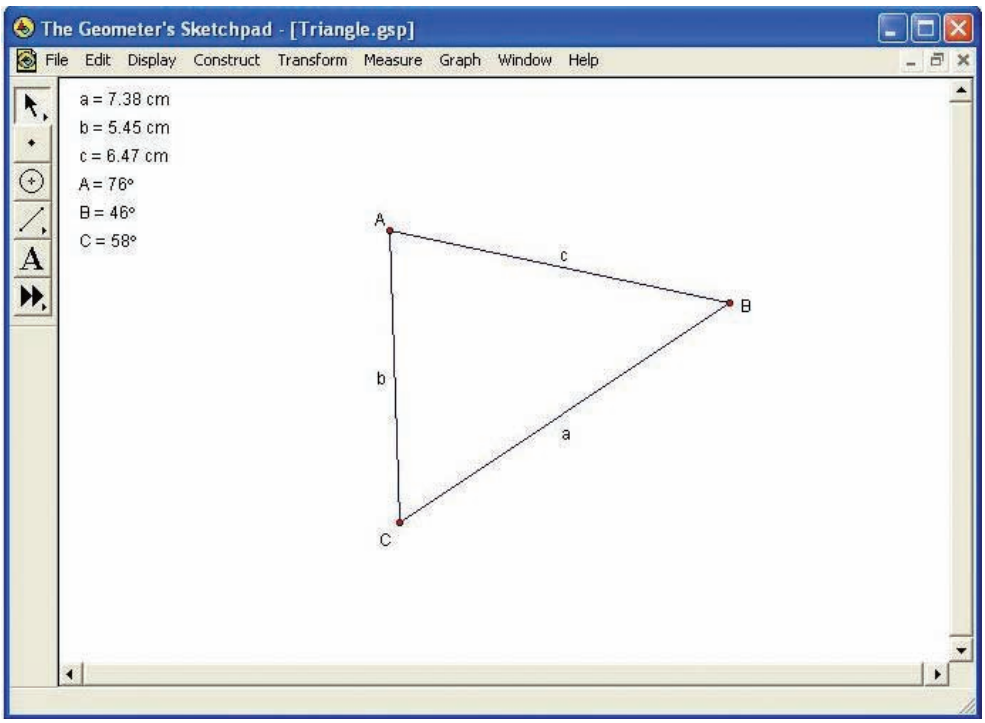
3. Use the **Selection Arrow** tool to select points A and B.
From the **Construct** menu, select **Segments**.
Use the **Text** tool.
Double click on line segment BC, and label it *a*.
Double click on line segment AC, and label it *b*.
Double click on line segment AB, and label it *c*.

4. Use the **Selection Arrow** tool to select side *a*.
From the **Measure** menu, select **Length**.
The value of *a* is displayed. What is the value of *a*?

5. Deselect the objects. Repeat question 4 for sides *b* and *c*.

Use the **Selection Arrow** tool and click on white space to deselect objects.

6. Use the **Selection Arrow** tool to select points B, A, and C, in this order.
From the **Measure** menu, select **Angle**.
The measure of ∠A will be displayed, labelled m∠BAC=.
To change the label, double click on it using the **Text** tool.
Enter A as the new label. Then, click **OK**.



7. Use the **Selection Arrow** tool to select points A, B, and C, in this order.
Repeat question 6 for the measure of ∠B.

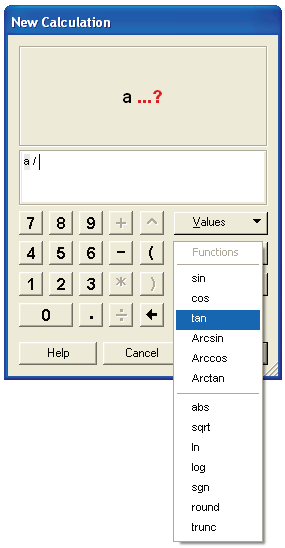
8. Select points B, C, and A, in this order.
Repeat question 6 for the measure of ∠C.
Use the **Selection Arrow** tool to drag points so that each angle in the triangle is acute.

9. What are the side lengths and angle measures of ΔABC?

Your screen should look like the one above, but with different side lengths and angle measures.

10. The **ratio** $\frac{a}{\tan A}$ compares the value of a to the tangent of the angle opposite a.
What do the ratios $\frac{b}{\tan B}$ and $\frac{c}{\tan C}$ compare?

11. From the **Measure** menu, select **Calculate**.
Click on the value of a, click on the $\div$ key,
select **tan** from the **Functions** list,
click on the measure of ∠A. Click on **OK**.
The value of $\frac{a}{\tan A}$ is displayed.
Follow a similar process to display $\frac{b}{\tan B}$ and $\frac{c}{\tan C}$.



12. What do the ratios $\frac{a}{\cos A}$, $\frac{b}{\cos B}$, and $\frac{c}{\cos C}$ compare?
Repeat question 11 to display these ratios.

13. What do the ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$ compare?
Repeat question 11 to display these ratios.

14. For which ratios are the values equivalent?

15. Drag one or more of the vertices of the triangle
to create a new acute triangle,
while keeping each angle less than 90°.
Are your observations about equivalent ratios
for the first triangle still true?
Repeat this process at least five more times.

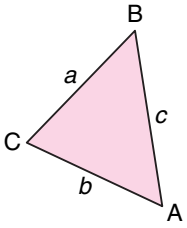
16. The ratio $\frac{\sin A}{a}$ compares the sine of ∠A to the value of the side opposite ∠A.
What do the ratios $\frac{\sin B}{b}$ and $\frac{\sin C}{c}$ compare?
Have the program calculate and display these ratios.
What do you notice?

17. Drag one or more of the vertices of the triangle to create a new acute triangle.
Are your observations from question 16 still true?

Making a Table

You will need a protractor and a scientific calculator.

1. Draw a large triangle with three acute angles.
Label the sides and angles of your triangle as shown.



An acute angle is less than 90°.

2. Measure the sides of your triangle to the nearest tenth of a centimetre.
Measure the angles of your triangle to the nearest degree.

3. Copy this table, and create six blank columns.
Record your data in the first six rows of one column.
Include data from five classmates in your table in the other columns.

Triangle drawn by:	
Length of a	
Length of b	
Length of c	
Measure of $\angle A$	
Measure of $\angle B$	
Measure of $\angle C$	
$\frac{a}{\tan A}$	
$\frac{b}{\tan B}$	
$\frac{c}{\tan C}$	
$\frac{a}{\cos A}$	
$\frac{b}{\cos B}$	
$\frac{c}{\cos C}$	
$\frac{a}{\sin A}$	
$\frac{b}{\sin B}$	
$\frac{c}{\sin C}$	

4. The ratio $\frac{a}{\tan A}$ compares the length of a to the tangent of the angle opposite a.
What do the ratios $\frac{b}{\tan B}$ and $\frac{c}{\tan C}$ compare?

5. What do the ratios $\frac{a}{\cos A}$, $\frac{b}{\cos B}$, and $\frac{c}{\cos C}$ compare?

6. What do the ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$ compare?

7. Record the ratios in questions 4, 5, and 6, to 3 decimal places in your table.

8. For which ratios are the values close?

9. The ratio $\frac{\sin A}{a}$ compares the sine of $\angle A$ to the value of a.
What do the ratios $\frac{\sin B}{b}$ and $\frac{\sin C}{c}$ compare?

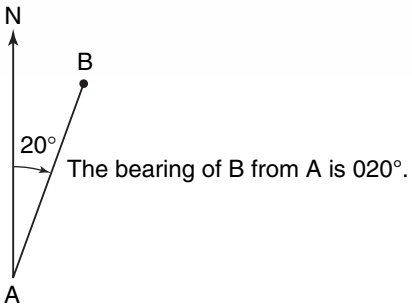
10. Calculate the ratios in question 9.
Enter the data, to 3 decimal places, in your table.
What do you notice about the values?

Reflect

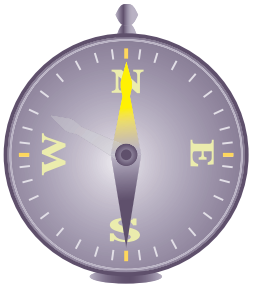
- The sine law shows this relationship for acute triangles:
  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
  How does your investigation verify the sine law?
- Does your investigation show a relationship for cosines or for tangents? Justify your answer.
- Did you use *The Geometer's Sketchpad* or a table?
  What is an advantage or disadvantage of this tool?

1.4 Applying the Sine Law

Navigation involves determining the position of a ship or aircraft and plotting its course.
In navigation, direction is often represented using a bearing. The bearing is given as a three-digit angle measured clockwise from the north line to a given direction.

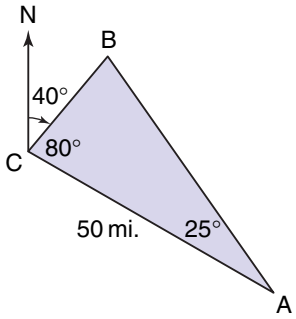


Investigate Using the Sine Law to Solve a Problem



Work with a partner.
You will need a scientific calculator.

The pilot of an ultralight plane, Carmen, is flying from Kingston, A, to Bancroft, B. Because of an error in compass readings, she flies off course by an angle of 25°. She travels 50 mi. before she realizes that she is going in the wrong direction.



Her new course is on a bearing of 040° to correct her flight path.
This triangle represents her situation.

- You know that $b = 50$ mi.
  How could you determine the measure of $\angle B$?
  What is its measure? Calculate $\frac{b}{\sin B}$.
- You know $\angle A = 25°$. What do you know about $\frac{a}{\sin A}$?
  How can you determine a? Calculate this length.
- How far is the plane from Bancroft, to the nearest mile, when Carmen realizes the error?

Reflect

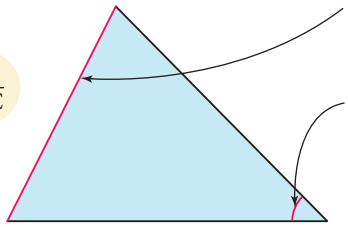
- Describe the strategy you used to solve the problem.
  Was it a reasonable strategy? Explain.
- How could you determine the distance from Kingston to Bancroft?

Connect the Ideas

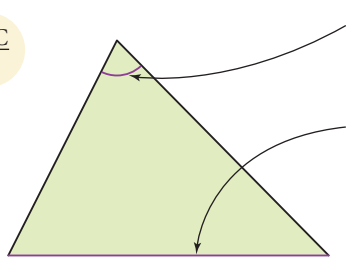
The sine law

The sine law relates the sides and angles in a triangle.
In any ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



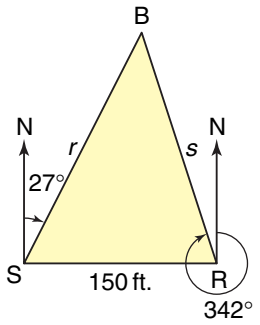
Saima is in a boat directly west of Richard's boat.
The distance between the boats is 150 ft.
Each person can see a buoy, B, from her or his boat.

The buoy indicates shallow water.

Saima sees the buoy on a bearing of 027°.
Richard sees it on a bearing of 342°.
How far is Saima's boat from the buoy, to the nearest foot?

Draw a diagram

Draw a triangle. Label its vertices S, R, and B to represent Saima, Richard, and the buoy.



We know the value of b. Determine ∠B.
In ΔSRB, the measure of ∠S is $90° - 27° = 63°$.

West is $3 \times 90° = 270°$ clockwise from north.
So, subtract 270° to determine the measure of ∠R in ΔSRB: $342° - 270° = 72°$
The measure of ∠B is $180° - 63° - 72° = 45°$.

The sum of the angles in a triangle is 180°.

Use the sine law

To determine the value of r, write the sine law for ΔSRB with r in the numerator.

$$\frac{r}{\sin R} = \frac{s}{\sin S} = \frac{b}{\sin B}$$

Substitute: ∠S = 63°, ∠R = 72°, ∠B = 45°, and $b = 150$

$$\frac{r}{\sin 72°} = \frac{s}{\sin 63°} = \frac{150}{\sin 45°}$$ Equate the 1st and 3rd ratios.

Solve the equation

$$\frac{r}{\sin 72°} = \frac{150}{\sin 45°}$$ Multiply each side by sin 72° to isolate r.

$$\frac{r}{\sin 72°} \times \sin 72° = \frac{150}{\sin 45°} \times \sin 72°$$

$$r = \frac{150 \sin 72°}{\sin 45°}$$ Press: 150 [×] [SIN] 72 [)] [÷] [SIN] 45 [)] [=]

$$r = 201.75$$

Saima's boat is about 202 ft. from the buoy.

Practice

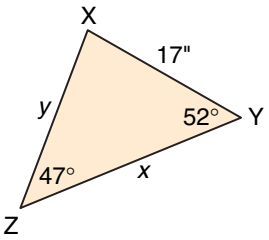
1. Calculate the value of c in each expression.
Give your answers to 1 decimal place.

a) $c = \frac{4 \sin 40°}{\sin 65°}$ **b)** $c = \frac{15 \sin 25°}{\sin 75°}$ **c)** $c = \frac{6.9 \sin 58°}{\sin 34°}$

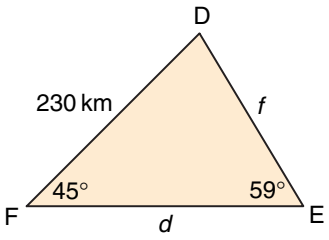
2. In *Connect the Ideas*, how far is Richard's boat from the buoy?

3. Determine the lengths of the unknown sides in each triangle.

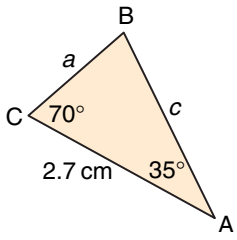
a)



b)

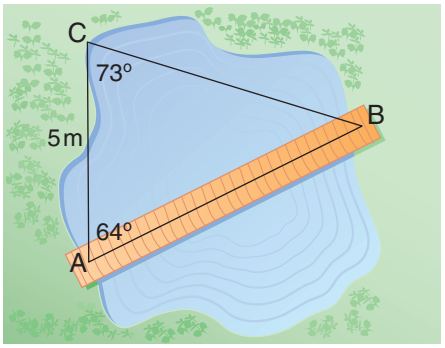


c)



4. Choose one triangle in question 3. Explain how you calculated the side lengths.

5. A landscape gardener plans a garden that includes a bridge over a small pond.
He marks the points on the shore where he wants the bridge to begin and end, A and B.
He marks another point on the shore, 5 m from one end of the bridge.
He measures the angles between the lines of sight joining the points.



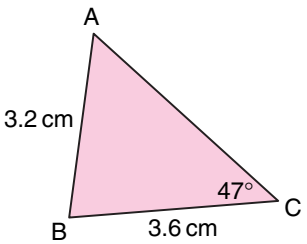
a) What is the distance across the pond, AB, that the bridge must span?

b) The landscape gardener can order prefabricated bridges that span 5.25 m, 7.25 m, or 9.25 m.
Which size of bridge should he order? Explain.

You have used the sine law to determine the side length of a triangle.
You can also use the sine law to determine the measure of an angle.

Example

What are the measures of the other two angles in this triangle, to the nearest degree?



Solution

The measure of one angle and the lengths of two sides are given.
One of the given sides is opposite the given angle.

Use the sine law: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Substitute: $a = 3.6$, $c = 3.2$, $\angle C = 47°$

$\frac{\sin A}{3.6} = \frac{\sin B}{b} = \frac{\sin 47°}{3.2}$ Equate the 1st and 3rd ratios.

$\frac{\sin A}{3.6} = \frac{\sin 47°}{3.2}$ Multiply each side by 3.6.

$\frac{\sin A}{3.6} \times 3.6 = \frac{\sin 47°}{3.2} \times 3.6$

$\sin A = \frac{\sin 47° \times 3.6}{3.2}$ Press: 2nd SIN SIN 47) × 3.6 ÷ 3.2) =

$\angle A \doteq 55.3633°$

The sum of the angles in a triangle is 180°.

$\angle B = 180° - 47° - 55.3633°$

$\angle B \doteq 77.64°$

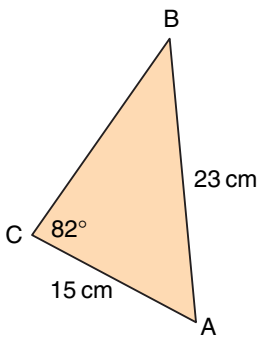
The measures of the other two angles are 55° and 78°, to the nearest degree.

6. In each expression, what is the measure of $\angle C$, to the nearest degree?

a) $\sin C = \frac{13 \sin 83°}{18}$ **b)** $\sin C = \frac{19 \sin 50°}{21}$ **c)** $\sin C = \frac{8.4 \sin 72°}{9.5}$

7. Determine the measures of the angles in each triangle, to the nearest degree.

a)



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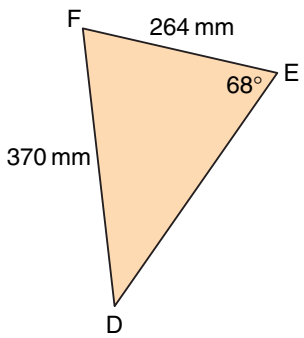
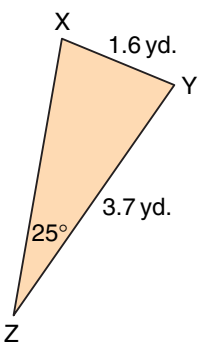
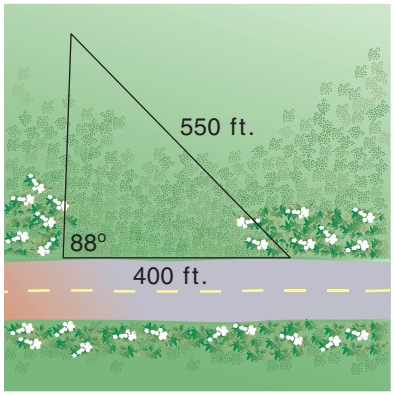


c)



8. Choose one triangle in question 7.
Describe how you calculated the angles.

9. **Assessment Focus** A surveyor has surveyed a triangular plot of land. One side of the plot lies along a county road. This side is 400 ft. long.
An adjacent side makes an angle of 88° with the side along the road. The side opposite this angle is 550 ft. long.



a) Determine the length of the third side of the plot, to the nearest foot.
b) Describe how you calculated the angles.
Explain whether your strategy was reasonable.

10. **Take It Further** Amanjeet is flying a kite in a flat, open field.
She lets out 15 m of string. The angle of elevation of the kite from her position is 64°.
Her friend Stephanie is also playing in the field, facing Amanjeet.
The angle of elevation of the kite from Stephanie's position is 35°.
a) Sketch a triangle using the given information.
b) How far apart are the girls, to the nearest metre?

In Your Own Words

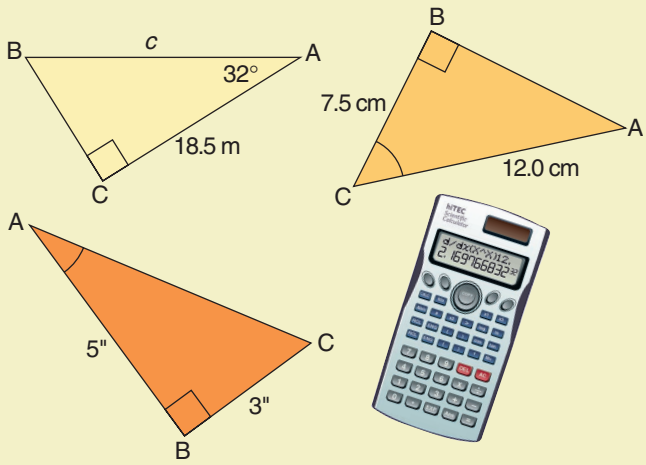
The measures of two angles in a triangle are known.
Suppose you want to determine the length of one side.
What additional information do you need to use the sine law?
Illustrate your answer with an example.
Identify three places after this section where you can find out more about the sine law.

Triangle Challenge

GAME

Materials

- triangle cards
- scientific calculators



Play in a group of 2 or 3.

➢ Shuffle the cards. Place them face down in a pile.

➢ Each player takes a card.
The player whose triangle shows the longest side length goes first.

➢ Replace the cards.
Shuffle again and place them face down in a pile.

Each card shows a triangle with some measures labelled, but one angle or side to be determined.

➢ The first player takes the top card from the pile
and places it face up.
He or she says what side or angle is to be determined.

➢ Each player determines that measure.

➢ Players compare answers.
If the player who turned over the card has the correct answer,
he or she keeps the triangle card.
If the player is not correct, the triangle card is returned
to the bottom of the pile.

Decide which rule you want to use for ending the game before you start playing.

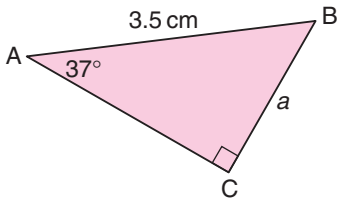
➢ The players take turns displaying cards from the pile.

➢ The game ends when each card in the pile
has been used once
or when there are no cards left in the pile.
The player with the most cards is the winner.

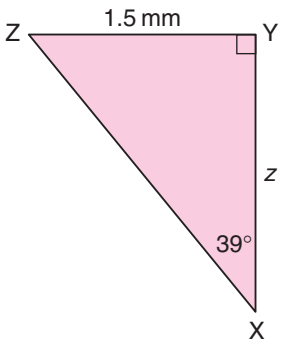
Mid-Chapter Review

1.1 **1.** Name the trigonometric ratio that can be used to calculate each length. Explain each choice. Then, calculate the lengths.

a)



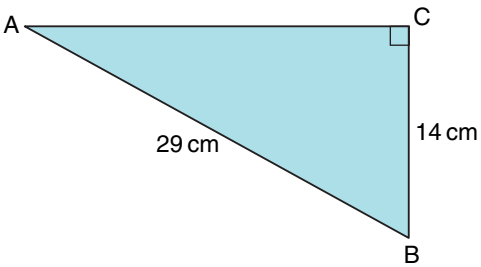
b)



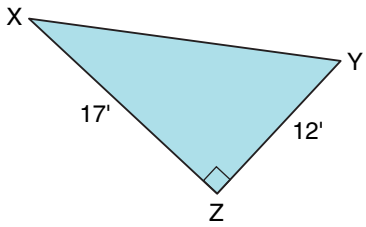
2. In 1966, the Falls Incline Railway opened in Niagara Falls.
It moves people from upper parking areas down to Queen Victoria Park.
The incline track is 170 ft. long and has an angle of inclination of 36°.
What is the vertical distance between the top station and the bottom station?

1.2 **3.** Determine the measure of the angle named. Explain your method.

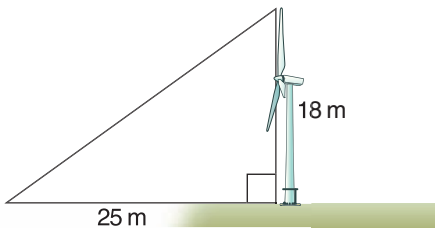
a) $\angle A$



b) $\angle X$

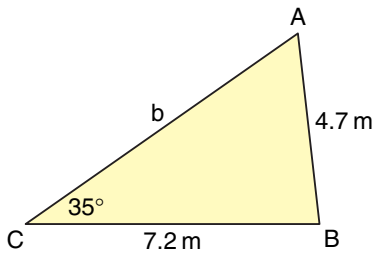


4. Simon measures the angle of elevation from the ground to the top of an 18-m high wind turbine.
He sets a clinometer on the ground 25 m from the base of the tower.
What is the angle of elevation?

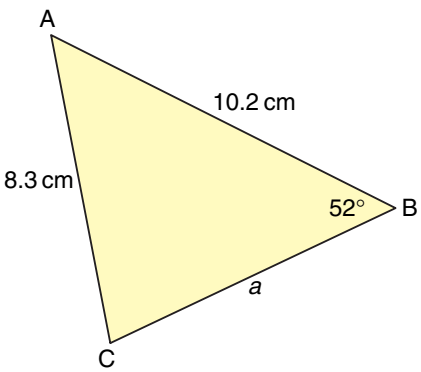


1.3 **5.** Determine the measure of $\angle A$.

1.4 a)



b)



1.5 The Cosine Law

A welder needs to cut a triangular shape from a piece of metal. He knows the lengths of the sides, but not the angle measures. He needs to know the angles.

The sine law cannot be used if no angles are known. Another relationship relating the sides and angles is needed.

Inquire Relating Sides and Angles in Triangles

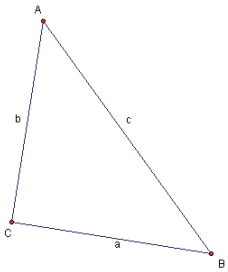
Choose Using *The Geometer's Sketchpad* or Making a Table.

Using The Geometer's Sketchpad

You will need *The Geometer's Sketchpad*.

- If you are using the file *Triangle2.gsp*, open it and begin at question 7.
- If you are not using the file *Triangle2.gsp*, complete all the questions.

1. Start the program.
From the **Edit** menu, select **Preferences**.
Set **Angle** to be measured in degrees to the nearest tenth.
Set **Distance** to be measured in centimetres to the nearest hundredth.



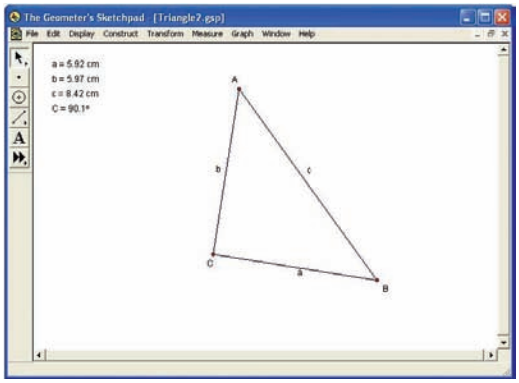
2. Use the **Point** tool.
Click on the screen in three places to construct three points.
Use the **Text** tool.
Click on each point to label the points A, B, and C.

3. Use the **Selection Arrow** tool to select points A and B.
From the **Construct** menu, select **Segment**.
Use the **Text** tool. Double click on line segment BC, and label it *a*.
Double click on line segment AC, and label it *b*.
Double click on line segment AB, and label it *c*.

Use the **Selection Arrow** tool and click on white space to deselect objects.

4. Use the **Selection Arrow** tool to select side *a*.
From the **Measure** menu, select **Length**.
The value of *a* will be displayed.

5. Deselect the objects. Repeat question 5 for sides *b* and *c*.



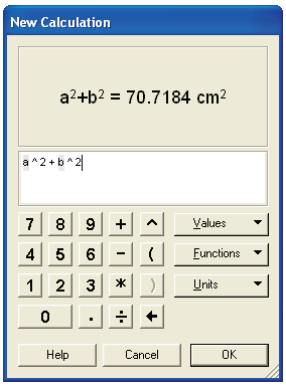
6. Use the **Selection Arrow** tool to select points A, C, and B, in this order. From the **Measure** menu, select **Angle**.
The measure of ∠C will be displayed, labelled m∠ACB=.
To change the label, double click on it using the **Text** tool.
Enter C as the new label. Then, click **OK**.

7. Use the **Selection Arrow** tool to drag point C so that ∠C is as close to 90° as possible.
From the **Measure** menu, select **Calculate**.
Click on the value of *a*, then click on the keys [^] 2 [+].
Click on the value of *b*, then click on the keys [^] 2. Click **OK**.
From the **Measure** menu, select **Calculate**.
Click on the value of *c*, then click on the keys [^] 2. Click **OK**.

Your screen should look like the one above, but with different side lengths and angle measures.

8. Does the calculation of $a^2 + b^2$ and c^2 fit the Pythagorean theorem for this triangle? Explain.

9. Drag a vertex of the triangle to change the values of *a* and *b* while keeping ∠C = 90°. Does the Pythagorean theorem still hold?



10. Drag point C so that ∠C is less than 90°.
Does the Pythagorean theorem still hold?
How do the values of $a^2 + b^2$ and c^2 compare?

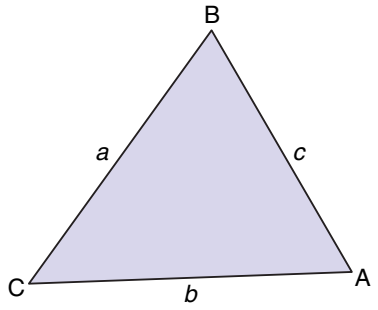
11. Drag a vertex of the triangle to change the values of *a* and *b* while keeping ∠C less than 90°.
How do the values of $a^2 + b^2$ and c^2 compare?

12. Use the **Calculate** feature to determine which expression results in the closest value to c^2 for your triangle.
$a^2 + b^2 - 2ab \sin C$ $a^2 + b^2 - 2ab \cos C$ $a^2 + b^2 - 2ab \tan C$

13. Change the size and shape of your triangle while keeping ∠C less than 90°.
Does the same expression give the result closest to c^2?

Making a Table

You will need a protractor and a scientific calculator.



1. Draw a triangle with three acute angles as large as possible.
   Label the sides and angles of your triangle as shown.

2. Measure the sides of your triangle to the nearest tenth of a centimetre.
   Measure $\angle C$ to the nearest degree.

3. Copy this table, and create six blank columns.
   Record your data in the first four rows.
   Include data from five classmates in your table.

Triangle drawn by:						
Length of a						
Length of b						
Length of c						
Measure of $\angle C$						
$a^2 + b^2$						
c^2						
$a^2 + b^2 - 2ab \sin C$						
$a^2 + b^2 - 2ab \cos C$						
$a^2 + b^2 - 2ab \tan C$						

4. Calculate the values for the next five rows of the table.

5. The Pythagorean Theorem states that if $\angle C = 90°$, $a^2 + b^2 = c^2$.
   In your triangle, $\angle C$ is less than 90°.
   Is the Pythagorean Theorem true for your triangle?
   What about for your classmates' triangles?

6. Look at the other expressions you calculated.
   Which is close to the value of c^2 for your triangle?
   Is this same relationship true for your classmates' triangles?

Reflect

- The cosine law shows this relationship:
  $c^2 = a^2 + b^2 - 2ab \cos C$
  How does your investigation verify the cosine law?
- Does your investigation show a relationship between c^2 and $a^2 + b^2 - 2ab \sin C$ or between c^2 and $a^2 + b^2 - 2ab \tan C$?
  Justify your answer.
- Did you use *The Geometer's Sketchpad* or a table?
  What is an advantage or a disadvantage of this tool?

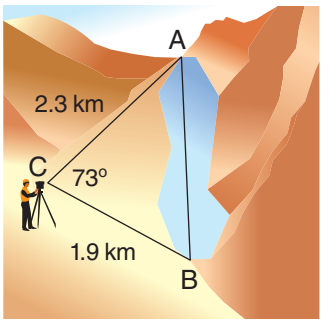
1.6 Applying the Cosine Law

Surveyors need to determine distances that cannot be measured directly.

Investigate Using the Cosine Law to Solve a Problem

You will need a scientific calculator.

A surveyor needs to determine the length of a gorge. She is standing at a point 2.3 km from the north end of the gorge. She is 1.9 km from the south end of the gorge. The surveyor uses a transit to measure the angle between the two lines of sight as 73°.



Discuss these questions with a partner.

- Which line segment represents the length of the gorge?
- Can you use the sine law to determine this length? Why or why not?
- Can you use the cosine law to determine the length of the gorge? Why or why not?
- On your own, determine the length of the gorge. Then, compare strategies and solutions with your partner.

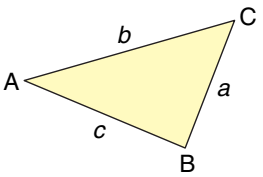
Reflect

- Explain your strategy for solving the problem.
- Are your results reasonable? How do you know?

Connect the Ideas

The cosine law

The cosine law relates the sides and angles in a triangle.



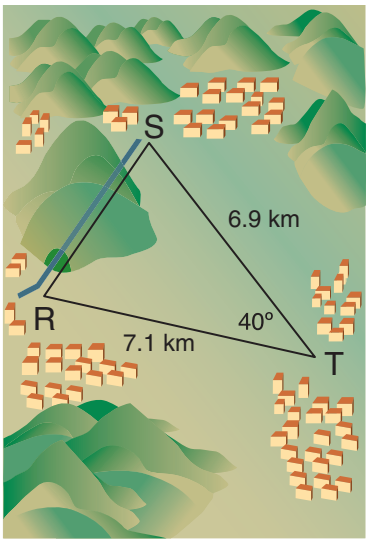
In any ΔABC,

$$c^2 = a^2 + b^2 - 2ab \times \cos C$$

The square of any side equals the sum of the squares of the other two sides minus (twice the product of these sides times the cosine of the angle between them.)

The towns of Rockcliff and Sutton are separated by a mountain. A straight railway tunnel is to be built joining the towns.

Rockcliff, R, is 7.1 km from Treyford, T. Sutton, S, is 6.9 km from Treyford. The angle between the lines joining each town to Treyford is 40°.



What is the distance between Rockcliff and Sutton, to the nearest tenth of a kilometre?

Use the cosine law

In ΔRST, you know the values of r and s and the measure of the angle between them, ∠T.
Determine the value of t.
Write the cosine law in terms of r, s, t, and ∠T.

$t^2 = r^2 + s^2 - 2rs \cos T$

Substitute: $r = 6.9$, $s = 7.1$, ∠T = 40°

$t^2 = 6.9^2 + 7.1^2 - 2 \times 6.9 \times 7.1 \times \cos 40°$

$t^2 \doteq 22.963$ Take the square root of each side to determine t.

Solve the equation

$\sqrt{t^2} = \sqrt{22.963}$

$t \doteq 4.79$

The tunnel will be about 4.8 km long.

Practice

Round angle measures to the nearest degree in your answers.

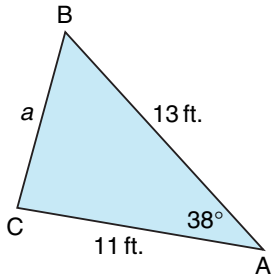
1. Identify the triangle with the unknown length that can be determined using each formula. Explain how you know the formula you chose is correct.

a) $c^2 = a^2 + b^2 - 2ab \cos C$

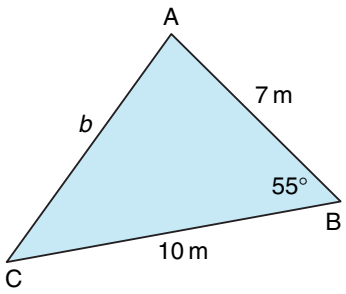
b) $b^2 = a^2 + c^2 - 2ac \cos B$

c) $a^2 = b^2 + c^2 - 2bc \cos A$

i)



ii)



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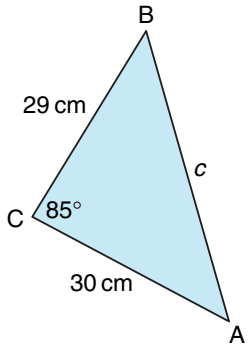
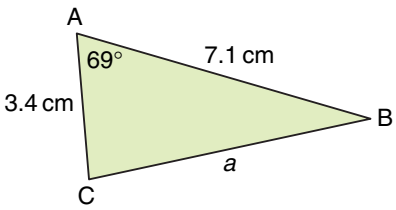
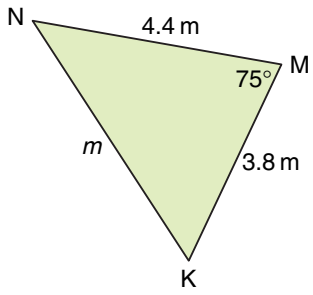


2. Write the formula you can use to determine the length of the unknown side in each triangle. Explain how you know your formula is correct.

a)



b)



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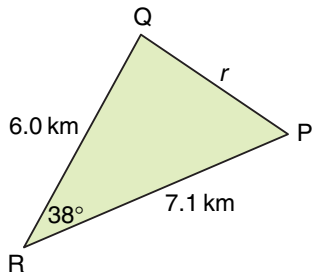
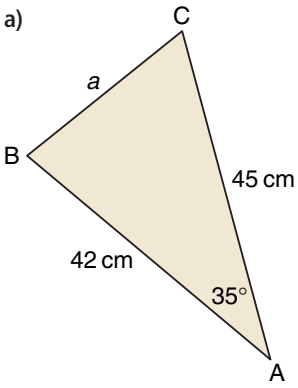


3. Determine the length of the unknown side in each triangle.

a)



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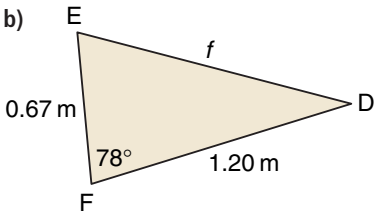

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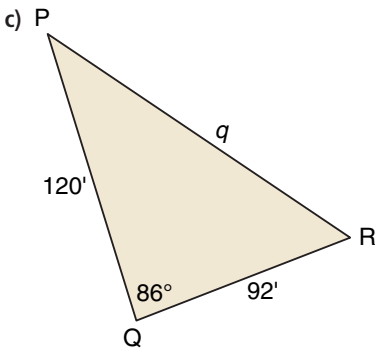
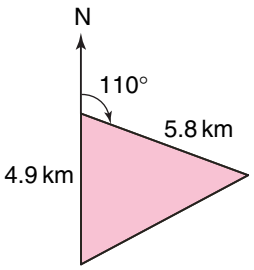


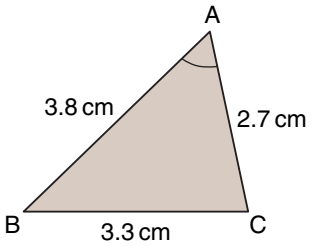
4. A hiker walked 4.9 km due north from her campsite. Then, she walked a further 5.8 km on a bearing of 110°. How far from the campsite is she now?



When you know the lengths of three sides of a triangle, you can use the cosine law to determine the measure of any angle in the triangle.

Example

Determine the measure of $\angle A$ in ΔABC to the nearest degree.



Solution

Use the cosine law formula with a^2 on the left and cos A on the right.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Substitute: $a = 3.3$, $b = 2.7$, $c = 3.8$

$$3.3^2 = 2.7^2 + 3.8^2 - (2 \times 2.7 \times 3.8 \cos A)$$

Remember the order of operations.

$10.89 = 21.73 - 28.08 \cos A$ Subtract 21.73 from each side.

$10.89 - 21.73 = 21.73 - 20.52 \cos A - 21.73$

$-10.84 = -20.52 \cos A$ Divide each side by -20.52.

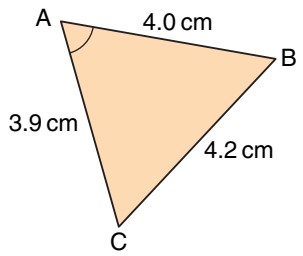
$\frac{-10.84}{-20.52} = \cos A$ Press: 2nd COS 10.84 ÷ 20.52) =

$\angle A \doteq 58.112°$

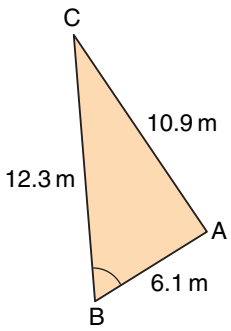
$\angle A$ is about 58°.

5. Write the formula you could use to determine the measure of the marked angle in each triangle.
Then, determine the angle measure.

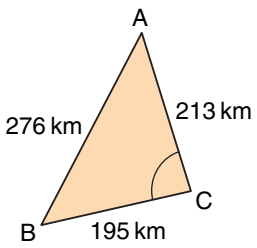
a) $\angle A$



b) $\angle B$



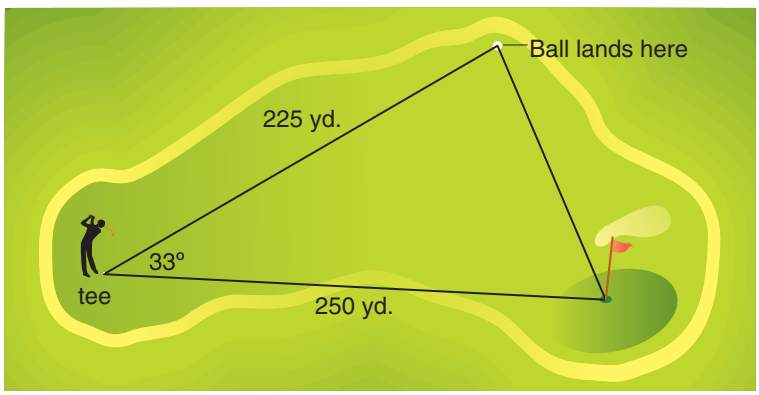
c) $\angle C$



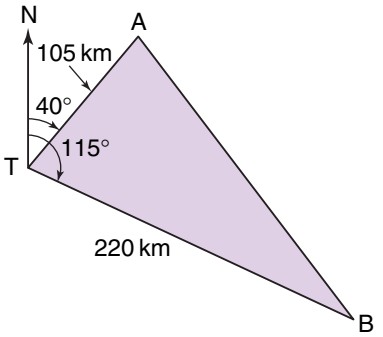
6. In the *Investigate*, suppose you knew the length of the gorge is 2.5 km, but did not know that $\angle C = 73°$.
How could you determine the measure of $\angle C$?

7. **a)** Create your own problem about the gorge in the *Investigate*.
b) Solve your problem.
c) Explain your strategy for creating your own problem.
d) Trade problems with a classmate. Solve the problems.

8. **Assessment Focus** A golfer hooks his first drive on a 250 yd. golf hole.
It lands 225 yd. from the tee, but is 33° off line, as shown in the diagram.
a) Determine how far from the hole the ball lands, to the nearest yard.
b) The golfer hits his next shot directly toward the hole. It travels 121 yd.
Will the ball reach the hole?
If so, how do you know? If not, how far from the hole will it be, to the nearest yard?
c) Explain how you know your solution is correct.



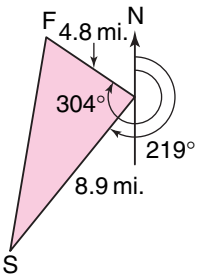
9. The diagram shows two airplanes approaching a control tower.
The first plane, A, is 105 km from the tower, T, on a bearing of 040°.
The second plane, B, is 220 km from the tower, T, on a bearing of 115°.



a) What is the measure of ∠ATB?

b) How far apart are the airplanes?

10. A ranger sights a forest fire. The fire, F, is 4.8 mi. from her observation tower, T, on a bearing of 304°.
She calls a forest protection service station, S.
The station is 8.9 mi. from the tower, on a bearing of 219°.



a) What is the measure of ∠FTS?

b) How far is the fire, F, from the station, S?

11. Take It Further How are the cosine law and the Pythagorean theorem the same? How are they different?
Include diagrams in your answer.

In Your Own Words

The lengths of two sides in a triangle are known.
What additional information do you need to use the cosine law?
Explain two possible solutions.
Describe two ways you can use this text to find out about the cosine law.

1.7 The Sine Law and the Cosine Law

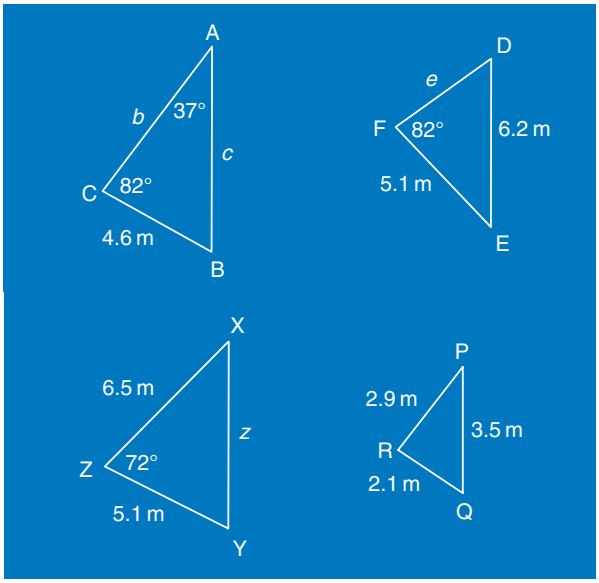
When solving a problem with an acute triangle, you may need to decide whether to use the sine law or the cosine law.

Investigate Choosing the Sine or Cosine Law

Work with a partner.
You will need a scientific calculator.

Alecia's plan for building a porch shows only some of measurements she needs for the pieces of wood.

- Alecia needs the length of b in ΔABC, the measure of $\angle D$ in ΔDEF, the length of z in ΔXYZ, and the measure of $\angle Q$ in ΔPQR.
- Decide whether she should use the sine law or the cosine law to determine each measurement. For each triangle, record the law you choose, entering the known measurements.



Reflect

- Justify each decision about which law to use.
- Is it necessary to solve for each measurement to know whether you chose the law Alecia can use? Explain.

Connect the Ideas

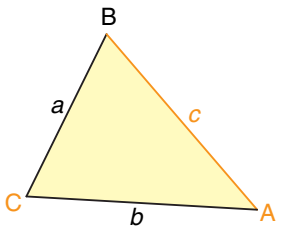
The sine law

In any ΔABC
Use the sine law when:

When you know the measures of two angles in a triangle, you can calculate the measure of the third angle. The sum of the angles in a triangle is 180°.

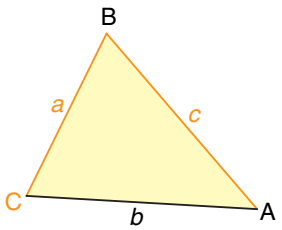
➢ you know the measures of two angles and the length of any side

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



➢ you know the lengths of two sides and the measure of an angle that is not between these sides

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

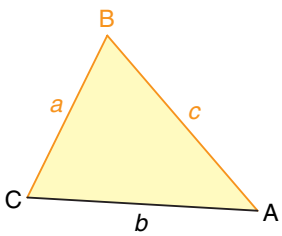


The cosine law

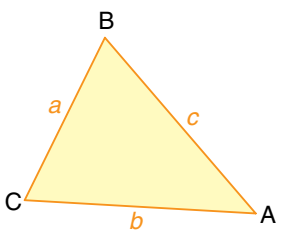
Use the cosine law when:

➢ you know the lengths of two sides and the measure of the angle between them

$$b^2 = a^2 + c^2 - 2ac \cos B$$



➢ you know the lengths of all three sides



Write the cosine law in a different way

You can solve $a^2 = b^2 + c^2 - 2bc \cos A$ for $\cos A$.
Then, determine the measure of an angle.

$a^2 = b^2 + c^2 - 2bc \cos A$ Add $2bc \cos A$ to each side.

$a^2 + 2bc \cos A = b^2 + c^2 - 2bc \cos A + 2bc \cos A$

$a^2 + 2bc \cos A = b^2 + c^2$ Subtract a^2 from each side.

$a^2 + 2bc \cos A - a^2 = b^2 + c^2 - a^2$

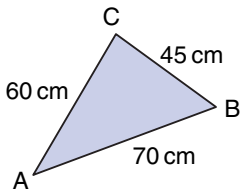
$2bc \cos A = b^2 + c^2 - a^2$ Divide each side by $2bc$.

$$\frac{2bc \cos A}{2bc} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

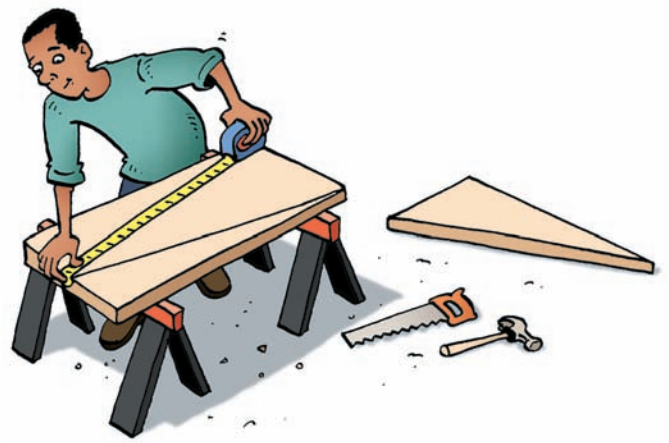
Note the position of A and *a* in the formula.

Max is designing a wooden plant stand with a triangular top. The side lengths of the triangle are 45 cm, 60 cm, and 70 cm. To make the triangle, Max needs to know the angle measures.



Choose the sine law or the cosine law

Since he knows three side lengths, he can use the cosine law to find any angle.



To determine the measure of ∠B, write the cosine law:

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

Substitute: $a = 70$, $c = 45$, and $b = 60$

$$\cos B = \frac{70^2 + 45^2 - 60^2}{2 \times 70 \times 45}$$

Press: (70 x^2 + 45 x^2 − 60 x^2) ÷ (2 × 70 × 45) =

$\cos B \doteq 0.5278$

$\angle B \doteq 58.1$

∠B is about 58°.

Practice

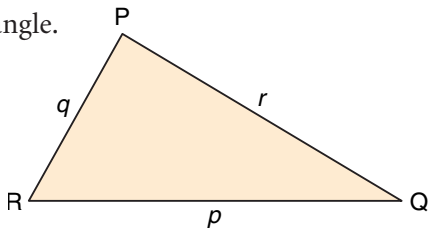
1. Determine the unknown measurements from the *Investigate*.

2. Write the equation you would use to determine the cosine of each angle in this triangle.

a) $\angle Q$

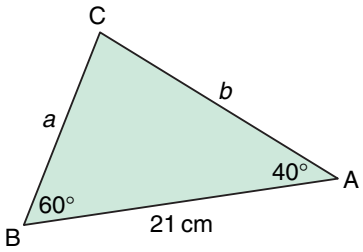
b) $\angle R$

c) $\angle P$

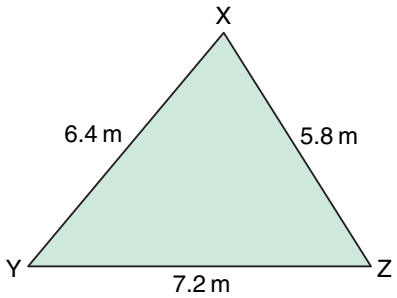


3. Decide whether to use the sine law or the cosine law to determine the indicated measure in each triangle. Justify your choices.

a) b

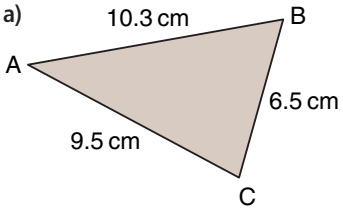


b) $\angle X$



4. Determine the measure of $\angle A$ for those triangles in which you can use the cosine law. Explain why you cannot use the cosine law in the other triangle. What other strategy could you use to find $\angle A$ in that triangle?

a)



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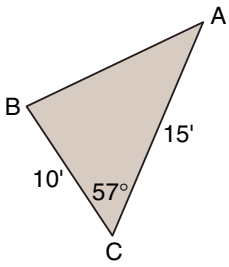

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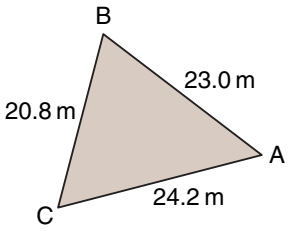
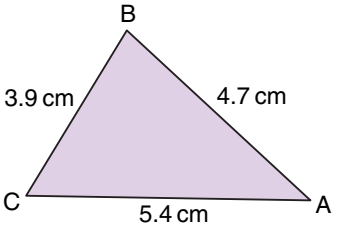
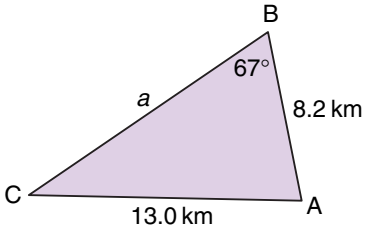


5. For each diagram, write an equation you can use to determine the unknown measure. Then calculate the measure.

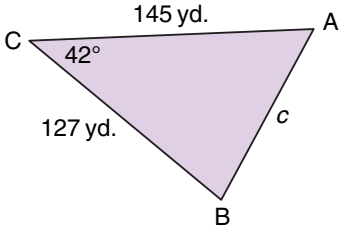
a) $\angle B$



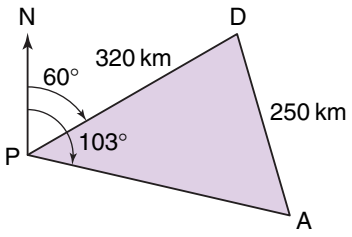
b) $\angle C$



c) c



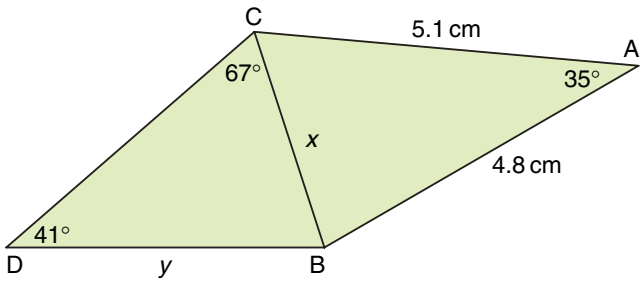
6. Assessment focus A plane, P, is 320 km from its destination, D.
Because of severe weather conditions, the plane is diverted to an airport, A, 250 km from the original destination, D.
The pilot changes his bearing from 060° to 103°.
How far must the plane fly to reach the airport, A?
Explain how you decided which law to use.



Sometimes you need to use more than one trigonometric law to solve a problem.
This is often true if two triangles are involved.

Example

Determine the values of x and y.



Solution

Use the cosine law to determine the value of x.

$a^2 = b^2 + c^2 - 2bc \cos A$

Substitute: $a = x$, $b = 5.1$, $c = 4.8$, $\angle A = 35°$

$x^2 = 5.1^2 + 4.8^2 - 2 \times 5.1 \times 4.8 \times \cos 35°$

$x^2 \doteq 8.9443$ Take the square root of each side to determine x.

Remember the order of operations.

$\sqrt{x^2} = \sqrt{8.9443}$

$x \doteq 2.99070$

To determine the value of y, use the sine law.

$$\frac{c}{\sin C} = \frac{d}{\sin D}$$

Substitute: $c = y$, $d = x = 2.9907$, $\angle C = 67°$, $\angle D = 41°$

$\frac{y}{\sin 67°} = \frac{2.9907}{\sin 41°}$ Multiply both sides by $\sin 67°$.

$\frac{y}{\sin 67°} \times \sin 67° = \frac{2.9907}{\sin 41°} \times \sin 67°$

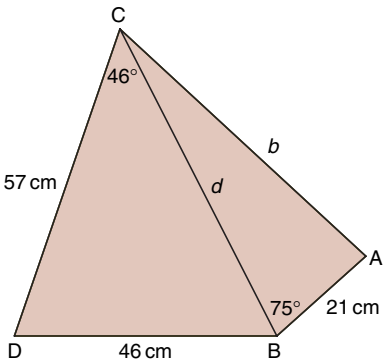
$y = \frac{2.9907}{\sin 41°} \times \sin 67°$

$y \doteq 4.196$

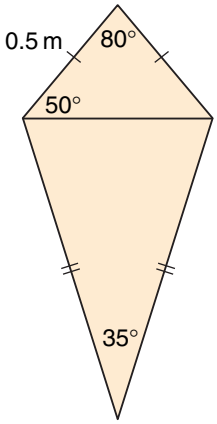
Round the value of x to 1 decimal place after calculating.

The value of x is about 3.0 cm and the value of y is about 4.2 cm.

7. Determine the measures of ∠CBD, ∠CDB, side d and side b.



8. Jaime's kite looks like this.



a) What is the width of the kite?

b) The two longest edges of the kite have the same length. Determine this length.

9. **Take It Further** Bogdan and Chloe are standing 70 ft. apart on opposite sides of the base of a lighthouse. The angle of elevation from each person to the top of the lighthouse is 37° from Bogdan and 56° from Chloe.

a) How tall is the lighthouse?

b) Explain the strategy you used to solve this problem.

In Your Own Words

Tom says he always uses the sine law to determine unknown measures because it is easier to use.

Amber says you can't use the sine law in every problem.

Who is correct? Explain how you would convince the other person.

Chapter Review

What Do I Need to Know?

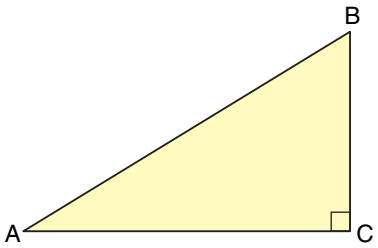
Primary Trigonometric Ratios: Sine, Cosine, and Tangent

For $\angle A$ in a right triangle,

$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$$

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$

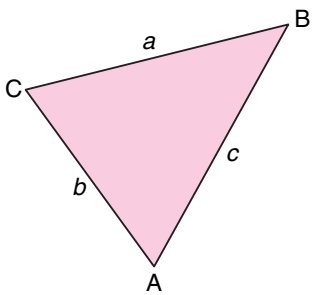


The Sine Law

In any acute ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



To use the sine law, you must know:

- the lengths of two sides and the measure of an angle opposite one of these sides or
- the measures of two angles and the length of any side

The Cosine Law

In any acute ΔABC,

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

To use the cosine law, you must know:

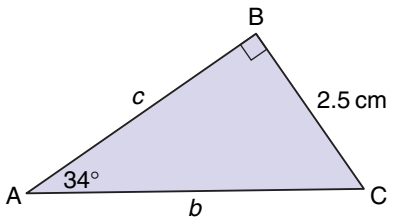
- the lengths of two sides and the measure of the angle between them or
- the lengths of all three sides

What Should I Be Able to Do?

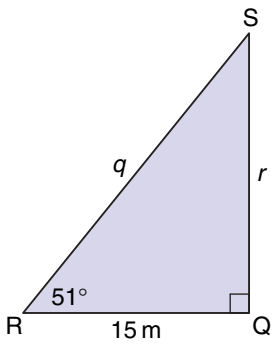
1.1

1. Calculate the measure of each unknown side. Explain your steps.

a)



b)



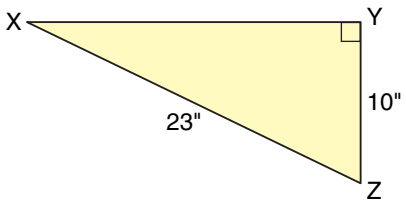
2. A tree casts a shadow 10 m long when the sun's rays make an angle of 25° with the ground.

a) Draw a diagram.

b) What is the height of the tree? Which trigonometric ratio did you use?

1.2

3. Calculate the measure of the unknown angles. Explain your steps.

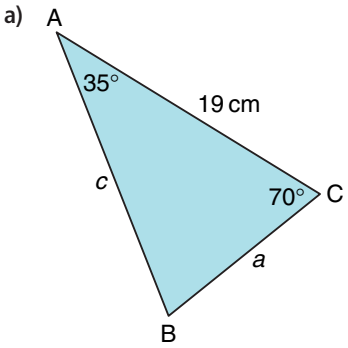


1.3

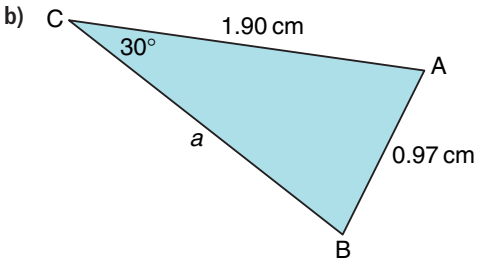
1.4

4. Determine the measures of the unknown sides and angles.

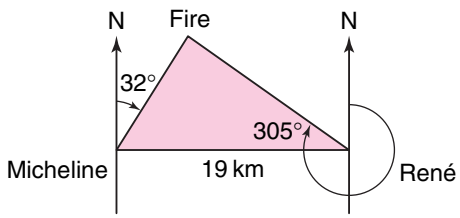
a)



b)



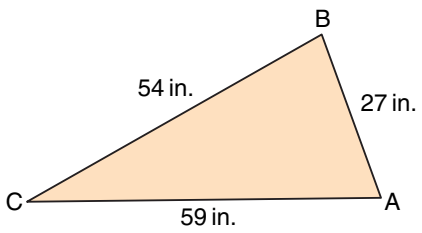
5. Two forest rangers, René and Micheline, observe the same fire from their observation towers. René's tower is 19 km due east of Micheline's. René sights the fire on a bearing of 305° from his tower. Micheline sights the fire on a bearing of 032° from her tower. How far is the fire from each tower?



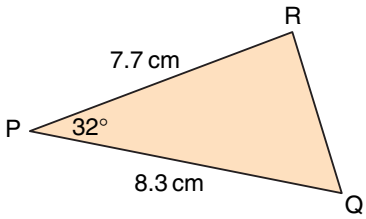
1.5 1.6

6. Determine the measures of the unknown sides and angles.

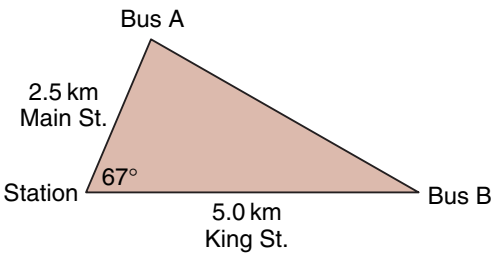
a)



b)



7. A bus station is at the intersection of Main St. and King St. The angle between the streets is 67°.
Two buses leave the bus station at the same time. After 15 min, one bus has travelled 2.5 km along Main St. The other bus has travelled 5.0 km along King St. How far apart are the buses?

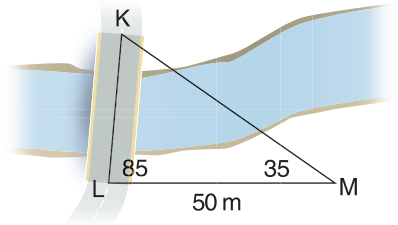


8. Determine the measure of the least angle in a triangle whose sides are 6 in., 7 in., and 9 in.

1.7

For each of questions 9 to 11, explain how you decided which law to use.

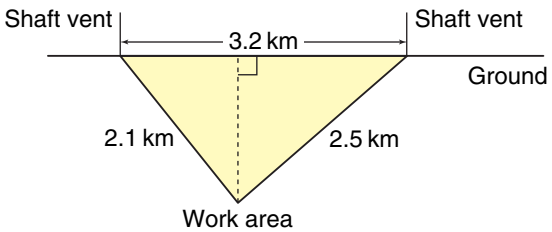
9. A bridge, KL, is to be built across a river. Point M is 50 m from L. ∠L is 85° and ∠M is 35°. How long will the bridge be, to the nearest metre?



10. A gold mine has two ventilation shafts that start at the same work area below ground. The vents of the shafts at ground level are 3.2 km apart. One shaft is 2.1 km long and the other is 2.5 km long.

a) What is the angle of depression of each shaft?

b) How far below ground is the work area?

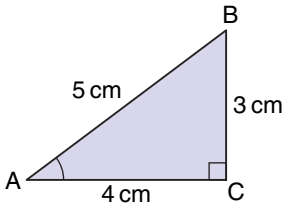


11. To measure the length of a glacier, a geographer stands at a point where she can see both ends of the glacier. The angle between the lines of sight from where she stands to the ends of the glacier is 62°. She measures the distance from where she stands to the ends to be 250 m and 215 m.
How long is the glacier?

Practice Test

Multiple Choice: Choose the correct answer for questions 1 and 2. Justify each choice.

1. Which ratio can you use to determine ∠A?

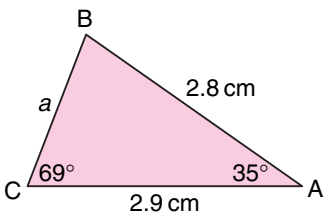


A. $\sin A = \frac{3}{4}$ **B.** $\sin A = \frac{3}{5}$ **C.** $\cos A = \frac{3}{5}$ **D.** $\tan A = \frac{5}{4}$

2. Which ratio can you use to determine the length of a?

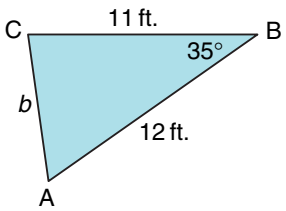
A. $\frac{a}{\sin 69°} = \frac{2.9}{\sin 35°}$ **B.** $\frac{a}{\sin 35°} = \frac{2.9}{\sin 69°}$

C. $\frac{a}{\sin 69°} = \frac{2.8}{\sin 35°}$ **D.** $\frac{a}{\sin 35°} = \frac{2.8}{\sin 69°}$



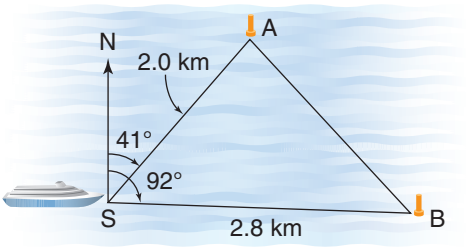
Show all your work for questions 3 to 6.

3. Knowledge and Understanding Determine the measure of side b to the nearest foot, and the measures of ∠A and ∠C to the nearest degree.



4. Communication Explain how you know when to use the sine law or the cosine law in an acute triangle.
Use an example for each law in your explanation.

5. Application A cruise ship sights channel marker A on a bearing of 041° and channel marker B on a bearing of 092°.
The distance from the ship to channel marker A is 2.0 km.
The distance from the ship to channel marker B is 2.8 km.
What is the distance between the channel markers?



6. Thinking The layout of a golf hole is shown in the diagram.
What is the distance from the tee directly to the hole?